Logic And Set Theory With Applications 6th Edition

This book deals with two important branches of mathematics, namely, logic and set theory. Logic and set theory are closely related and play very crucial roles in the foundation of mathematics, and together produce several results in all of mathematics. The topics of logic and set theory are required in many areas of physical sciences, engineering, and technology. The book offers solved examples and exercises, and provides reasonable details to each topic discussed, for easy understanding. The book is designed for readers from various disciplines where mathematical logic and set theory play a crucial role. The book will be of interested to students and instructors in engineering, mathematics, computer science, and technology.

A mathematical introduction to the theory and applications of logic and set theory with an emphasis on writing proofs Highlighting the applications and notations of basic mathematical concepts within the framework of logic and set theory, A First Course in Mathematical Logic and Set Theory introduces how logic is used to prepare and structure proofs and solve more complex problems. The book begins with propositional logic, including two-column proofs and truth table applications, followed by first-order logic, which provides the structure for writing mathematical proofs. Set theory is then introduced and serves as the basis for defining relations, functions, numbers, mathematical induction, ordinals, and cardinals. The book concludes with a primer on basic model theory with applications to abstract algebra. A First Course in Mathematical Logic and Set Theory also includes: Section exercises designed to show the interactions between topics and reinforce the presented ideas and concepts Numerous examples that illustrate theorems and employ basic concepts such as Euclid's lemma, the Fibonacci sequence, and unique factorization Coverage of important theorems including the well-ordering theorem, completeness theorem, compactness theorem, as well as the theorems of Löwenheim–Skolem, Burali-Forti, Hartogs, Cantor–Schröder–Bernstein, and König An excellent textbook for students studying the foundations of mathematics and mathematical proofs, A First Course in Mathematical Logic and Set Theory is also appropriate for readers preparing for careers in mathematics education or computer science. In addition, the book is ideal for introductory courses on mathematical logic and/or set theory and appropriate for upper-undergraduate transition courses with rigorous mathematical reasoning involving algebra, number theory, or analysis.

This short textbook provides a succinct introduction to mathematical logic and set theory, which together form the foundations for the rigorous development of mathematics. It will be suitable for all mathematics undergraduates coming to the subject for the first time. The book is based on lectures given at the University of Cambridge and covers the basic concepts of logic: first order logic, consistency, and the completeness theorem, before introducing the reader to the fundamentals of axiomatic set theory. There are also chapters on recursive functions, the axiom of choice, ordinal and cardinal arithmetic and the incompleteness theorems. Dr Johnstone has included numerous exercises designed to illustrate the key elements of the theory and to provide applications of basic logical concepts to other areas of mathematics. Consequently the book, while making an attractive first textbook for those who plan to specialise in logic, will be particularly valuable for mathematics and computer scientists whose primary interests lie elsewhere.

This lively introductory text exposes the student in the humanities to the world of discrete mathematics. A problem-solving based approach grounded in the ideas of George POlya are at the heart of this book. Students learn to handle and solve new problems on their own. A straightforward, clear writing style and well-crafted examples with diagrams invite the students to develop into precise and critical thinkers. Particular attention has been given to the material that some students find challenging, such as proofs. This book illustrates how to spot invalid arguments, to enumerate possibilities, and to construct probabilities. It also presents case studies to students about the possible detrimental effects of ignoring these basic principles. The book is invaluable for a discrete and finite mathematics course at the freshman undergraduate level or for self-study since there are full solutions to the exercises in an appendix. "Written with clarity, humor and relevant real-world examples, Basic Discrete Mathematics is a wonderful introduction to discrete mathematical reasoning." - Arthur Benjamin, Professor of Mathematics at Harvey Mudd College, and author of The Magic of Math

Logic, Set Theory, & Probability

Theorems, Philosophies

Transl. by O. Wojtasiewicz

An Introduction to Abstract Mathematics, Third Edition

Combinatorial Set Theory of C*-algebras

The authors offer a clear, succinct and basic introduction to set theory and formal logic for linguists.

A new approach to the standard axioms of set theory, relating the theory to the philosophy of science and metametaphysics.

What is a number? What is infinity? What is continuity? What is order? Answers to these fundamental questions obtained by late nineteenth-century mathematicians such as Dedekind and Cantor gave birth to set theory. This textbook presents classical set theory in an intuitive but concrete manner. To allow flexibility of topic selection in courses, the book is organized into four relatively independent parts with distinct mathematical flavors. Part I begins with the Dedekind–Peano axioms and ends with the construction of the real numbers. The core Cantor–Dedekind theory of cardinals, orders, and ordinals appears in Part II. Part III focuses on the real continuum. Finally, foundational issues and formal axioms are introduced in Part IV. Each part ends with a postscript chapter discussing topics beyond the scope of the main text, ranging from philosophical remarks to glimpses into landmark results of modern set theory such as the resolution of Lusin's problems on projective sets using determinacy of infinite games and large cardinals. Separating the metamathematical issues into an optional fourth part at the end makes this textbook suitable for students interested in any field of mathematics, not just for those planning to specialize in logic or foundations. There is enough material in the text for a year-long course at the upper-undergraduate level. For shorter one-semester or one-quarter courses, a variety of arrangements of topics are possible. The book will be a useful resource for both experts working in a relevant or adjacent area and beginners wanting to learn set theory via self-study.

Set theory is a branch of mathematics with a special subject matter, the infinite, but also a general framework for all modern mathematics, whose notions figure in every branch, pure and applied. This Element will offer a concise introduction, treating the origins of the subject, the basic notion of set, the axioms of set theory and immediate consequences, the set-theoretic reconstruction of mathematics, and the theory of the infinite, touching also on selected topics from higher set theory, controversial axioms and undecided questions, and philosophical issues raised by technical developments.

Lectures in Logic and Set Theory

Applying Formalized Logic to Analysis

Elements of Set Theory

Classical Descriptive Set Theory

With an Introduction to Real Point Sets

Set Theory and LogicCourier Corporation

By its nature, set theory does not depend on any previous mathematical knowl edge. Hence, an individual wanting to read this book can best find out if he is ready to do so by trying to read the first ten or twenty pages of Chapter 1. As a textbook, the book can serve for a course at the junior or senior level. If a course covers only some of the chapters, the author hopes that the student will read the rest himself in the next year or two. Set theory has always been a sub ject which people find pleasant to study at least partly by themselves. Chapters 1-7, or perhaps 1-8, present the core of the subject. (Chapter 8 is a short, easy discussion of the axiom of regularity). Even a hurried course should try to cover most of this core (of which more is said below). Chapter 9 presents the logic needed for a fully axiomatic set th~ory and especially for independence or consistency results. Chapter 10 gives von Neumann's proof of the relative consistency of the regularity axiom and three similar related results. Von Neumann's 'inner model' proof is easy to grasp and yet it prepares one for the famous and more difficult work of GOdel and Cohen, which are the main topics of any book or course in set theory at the next level.

"This accessible approach to set theory for upper-level undergraduates poses rigorous but simple arguments. Each definition is accompanied by commentary that motivates and explains new concepts. A historical introduction is followed by discussions of classes and sets, functions, natural and cardinal numbers, the arithmetic of ordinal numbers, and related topics. 1971 edition with new material by the author"--

Problems in Set Theory, Mathematical Logic and the Theory of Algorithms by I. Lavrov & L. Maksimova is an English translation of the fourth edition of the most popular student problem book in mathematical logic in Russian. It covers major classical topics in proof theory and the semantics of propositional and predicate logic as well as set theory and computation theory. Each chapter begins with 1-2 pages of terminology and definitions that make the book self-contained. Solutions are provided. The book is likely to become an essential part of curricula in logic.

Set Theory

Set Theory: An Introduction

A First Course

Computational Logic and Set Theory

Set theory can be considered a unifying theory for mathematics. This book covers the fundamentals of the subject.

This collection of papers from various areas of mathematical logic showcases the remarkable breadth and richness of the field. Leading authors reveal how contemporary technical results touch upon foundational questions about the nature of mathematics. Highlights of the volume include: a history of Tennenbaum's theorem in arithmetic; a number of papers on Tennenbaum phenomena in weak arithmetics as well as on other aspects of arithmetics, such as interpretability; the transcript of Gödel's previously unpublished 1972–1975 conversations with Sue Toledo, along with an appreciation of the same by Curtis Franks; Hugh Woodin's paper arguing against the generic multiverse view; Anne Troelstra's history of intuitionism through 1991; and Aki Kanamori's history of the Suslin problem in set theory. The book provides a historical and philosophical treatment of particular theorems in arithmetic and set theory, and is ideal for researchers and graduate students in mathematical logic and philosophy of mathematics.

Volume II, on formal (ZFC) set theory, incorporates a self-contained "chapter 0" on proof techniques so that it is based on formal logic, in the style of Bourbaki. The emphasis on basic techniques provides a solid foundation in set theory and a thorough context for the presentation of advanced topics (such as absoluteness, relative consistency results, two expositions of Godel's construstive universe, numerous ways of viewing recursion and Cohen forcing).

This is an extensively revised edition of Mr. Quine's introduction to abstract set theory and to various axiomatic systematizations of the subject. The treatment of ordinal numbers has been strengthened and much simplified, especially in the theory of transfinite recursions, by adding an axiom and reworking the proofs. Infinite cardinals are treated anew in clearer and fuller terms than before. Improvements have been made all through the book; in various instances a proof has been shortened, a theorem strengthened, a space-saving lemma inserted, an obscurity clarified, an error corrected, a historical omission supplied, or a new event noted.

Logic, Set Theory, and Probability

Notes on Logic and Set Theory

Categories for the Working Philosopher

Logic in Linguistics

With an Introduction to Computer Programming

This book is intended as an undergraduate senior level or beginning graduate level text for mathematical logic. There are virtually no prere quisites, although a familiarity with notions encountered in a beginning course in abstract algebra such as groups, rings, and fields will be useful in providing some motivation for the topics in Part III. An attempt has been made to develop the beginning of each part slowly and then to gradually quicken the pace and the complexity of the material. Each part ends with a brief introduction to selected topics of current interest. The text is divided into three parts: one dealing with set theory, another with computable function theory, and the last with model theory. Part III relies heavily on the notation, concepts and results discussed in Part I and to some extent on Part II. Parts I and II are independent of each other, and each provides enough material for a one semester course. The exercises cover a wide range of difficulty with an emphasis on more routine problems in the earlier sections of each part in order to familiarize the reader with the new notions and methods. The more difficult exercises are accompanied by hints. In some cases significant theorems are devel oped step by step with hints in the problems. Such theorems are not used later in the sequence.

Keith Devlin. You know him. You've read his columns in MAA Online, you've heard him on the radio, and you've seen his popular mathematics books. In between all those activities and his own research, he's been hard at work revising Sets, Functions and Logic, his standard-setting text that has smoothed the road to pure mathematics for legions of undergraduate students. Now in its third edition, Devlin has fully reworked the book to reflect a new generation. The narrative is more lively and less textbook-like. Remarks and asides link the topics presented to the real world of students' experience. The chapter on complex numbers and the discussion of formal symbolic logic are gone in favor of more exercises, and a new introductory chapter on the nature of mathematics--one that motivates readers and sets the stage for the challenges that lie ahead. Students crossing the bridge from calculus to higher mathematics need and deserve all the help they can get. Sets, Functions, and Logic, Third Edition is an affordable little book that all of your transition-course students not only can afford, but will actually read...and enjoy...and learn from. About the Author Dr. Keith Devlin is Executive Director of Stanford University's Center for the Study of Language and Information and a Consulting Professor of Mathematics at Stanford. He has written 23 books, one interactive book on CD-ROM, and over 70 published research articles. He is a Fellow of the American Association for the Advancement of Science, a World Economic Forum Fellow, and a former member of the Mathematical Sciences Education Board of the National Academy of Sciences,. Dr. Devlin is also one of the world's leading popularizers of mathematics. Known as "The Math Guy" on NPR's Weekend Edition, he is a frequent contributor to other local and national radio and TV shows in the US and Britain, writes a monthly column for the Web journal MAA Online, and regularly writes on mathematics and computers for the British newspaper The Guardian.

This must-read text presents the pioneering work of the late Professor Jacob (Jack) T. Schwartz on computational logic and set theory and its application to proof verification techniques, culminating in the ÆtnaNova system, a prototype computer program designed to verify the correctness of mathematical proofs presented in the language of set theory. Topics and features: describes in depth how a specific first-order theory can be exploited to model and carry out reasoning in branches of computer science and mathematics; presents an unique system for automated proof verification in large-scale software systems; integrates important proof-engineering issues, reflecting the goals of large-scale verifiers; includes an appendix showing formalized proofs of ordinals, of various properties of the transitive closure operation, of finite and transfinite induction principles, and of Zorn's lemma.

This book explores and highlights the fertile interaction between logic and operator algebras, which in recent years has led to the resolution of several long-standing open problems on C*-algebras. The interplay between logic and operator algebras (C*-algebras, in particular) is relatively young and the author is at the forefront of this interaction. The deep level of scholarship contained in these pages is evident and opens doors to operator algebraists interested in learning about the set-theoretic methods relevant to their field, as well as to set-theorists interested in expanding their view to the non-commutative realm of operator algebras. Enough background is included from both subjects to make the book a convenient, self-contained source for students. A fair number of the exercises form an integral part of the text. They are chosen to widen and deepen the material from the corresponding chapters. Some other exercises serve as a warmup for the latter chapters.

Set Theory, Arithmetic, and Foundations of Mathematics

Introduction to Mathematical Logic

Logic and Set Theory

With Logic and Set Theory

Sets, Logic and Categories

Set theory, logic and category theory lie at the foundations of mathematics, and have a dramatic effect on the mathematics that we do, through the Axiom of Choice, Gödel's Theorem, and the Skolem Paradox. But they are also rich mathematical theories in their own right, contributing techniques and results to working mathematicians such as the Compactness Theorem and module categories. The book is aimed at those who know some mathematics and want to know more about its building blocks. Set theory is first treated naively an axiomatic treatment is given after the basics of first-order logic have been introduced. The discussion is su pported by a wide range of exercises. The final chapter touches on philosophical issues. The book is supported by a World Wibe Web site containing a variety of supplementary material.

This is the first volume on category theory for a broad philosophical readership. It is designed to show the interest and significance of category theory for a range of philosophical interests: mathematics, proof theory, computation, cognition, scientific modelling, physics, ontology, the structure of the world. Each chapter is written by either a category-theorist or a philosopher working in one of the represented areas, in an accessible waythat builds on the concepts that are already familiar to philosophers working in these areas.

Explores sets and relations, the natural number sequence and its generalization, extension of natural numbers to real numbers, logic, informal axiomatic mathematics, Boolean algebras, informal axiomatic set theory, several algebraic theories, and 1st-order theories.

Gert H. Muller The growth of the number of publications in almost all scientific areas, as in the area of (mathematical) logic, is taken as a sign of our scientifically minded culture, but it also has a terrifying aspect. In addition, given the rapidly growing sophistica tion, specialization and hence subdivision of logic, researchers, students and teachers may have a hard time getting an overview of the existing literature, partic ularly if they do not have an extensive library available in their neighbourhood: they simply do not even know what to ask for! More specifically, if someone vaguely knows that something vaguely connected with his interests exists some where in the literature, he may not be able to find it even by searching through the publications scattered in the review journals. Answering this challenge was and is the central motivation for compiling this Bibliography. The Bibliography comprises (presently) the following six volumes (listed with the corresponding Editors): I. Classical Logic W. Rautenberg II. Non-classical Logics W. Rautenberg III. Model Theory H. -D. Ebbinghaus IV. Recursion Theory P. G. Hinman V. Set Theory A. R. Blass VI. ProofTheory; Constructive Mathematics J. E. Kister; D. van Dalen & A. S. Troelstra.

Set Theory Computable Functions Model Theory

Set Theory, Logic and Their Limitations

Logic and Set Theory with Applications

Omega-Bibliography of Mathematical Logic V

Logic for Physicists

The main body of this book consists of 106 numbered theorems and a dozen of examples of models of set theory. A large number of additional results is given in the exercises, which are scattered throughout the text. Most exer cises are provided with an outline of proof in square brackets [ ], and the more difficult ones are indicated by an asterisk. I am greatly indebted to all those mathematicians, too numerous to men tion by name, who in their letters, preprints, handwritten notes, lectures, seminars, and many conversations over the past decade shared with me their insight into this exciting subject. XI CONTENTS Preface xi PART I SETS Chapter 1 AXIOMATIC SET THEORY I. Axioms of Set Theory I 2. Ordinal Numbers 12 3. Cardinal Numbers 22 4. Real Numbers 29 5. The Axiom of Choice 38 6. Cardinal Arithmetic 42 7. Filters and Ideals. Closed Unbounded Sets 52 8. Singular Cardinals 61 9. The Axiom of Regularity 70 Appendix: Bernays-Godel Axiomatic Set Theory 76 Chapter 2 TRANSITIVE MODELS OF SET THEORY 10. Models of Set Theory 78 II. Transitive Models of ZF 87 12. Constructible Sets 99 13. Consistency of the Axiom of Choice and the Generalized Continuum Hypothesis 108 14. The In Hierarchy of Classes, Relations, and Functions 114 15. Relative Constructibility and Ordinal Definability 126 PART II MORE SETS Chapter 3 FORCING AND GENERIC MODELS 16. Generic Models 137 17. Complete Boolean Algebras 144 18.

This book is designed for readers who know elementary mathematical logic and axiomatic set theory, and who want to learn more about set theory. The primary focus of the book is on the independence proofs. Most famous among these is the independence of the Continuum Hypothesis (CH); that is, there are models of the axioms of set theory (ZFC) in which CH is true, and other models in which CH is false. More generally, cardinal exponentiation on the regular cardinals can consistently be anything not contradicting the classical theorems of Cantor and König. The basic methods for the independence proofs are the notion of constructibility, introduced by Gödel, and the method of forcing, introduced by Cohen. This book describes these methods in detail, verifi es the basic independence results for cardinal exponentiation, and also applies these methods to prove the independence of various mathematical questions in measure theory and general topology. Before the chapters on forcing, there is a fairly long chapter on "infi nitary combinatorics". This consists of just mathematical theorems (not independence results), but it stresses the areas of mathematics where set-theoretic topics (such as cardinal arithmetic) are relevant. There is, in fact, an interplay between infi nitary combinatorics and independence proofs. Infi nitary combinatorics suggests many set-theoretic questions that turn out to be independent of ZFC, but it also provides the basic tools used in forcing arguments. In particular, Martin's Axiom, which is one of the topics under infi nitary combinatorics, introduces many of the basic ingredients of forcing.

This is an introductory undergraduate textbook in set theory. In mathematics these days, essentially everything is a set. Some knowledge of set theory is necessary part of the background everyone needs for further study of mathematics. It is also possible to study set theory for its own interest--it is a subject with intruiging results anout simple objects. This book starts with material that nobody can do without. There is no end to what can be learned of set theory, but here is a beginning.

This book gives a rigorous yet 'physics-focused' introduction to mathematical logic that is geared towards natural science majors. We present the science major with a robust introduction to logic, focusing on the specific knowledge and skills that will unavoidably be needed in calculus topics and natural science topics in general (rather than taking a philosophical math fundamental oriented approach that is commonly found in mathematical logic textbooks).

Lectures in Logic and Set Theory: Volume 2, Set Theory

Concise Introduction to Logic and Set Theory

Elements of mathematical logic and set theory

Set Theory and Logic

A Logical Foundation for Potentialist Set Theory

Rigorous coverage of logic and set theory for students of mathematics and philosophy.

This lively introductory text exposes the student in the humanities to the world of discrete mathematics. A problem-solving based approach grounded in the ideas of George Pólya are at the heart of this book. Students learn to handle and solve new problems on their own. A straightforward, clear writing style and well-crafted examples with diagrams invite the students to develop into precise and critical thinkers. Particular attention has been given to the material that some students find challenging, such as proofs. This book illustrates how to spot invalid arguments, to enumerate possibilities, and to construct probabilities. It also presents case studies to students about the possible detrimental effects of ignoring these basic principles. The book is invaluable for a discrete and finite mathematics course at the freshman undergraduate level or for self-study since there are full solutions to the exercises in an appendix. "Written with clarity, humor and relevant real-world examples, Basic Discrete Mathematics is a wonderful introduction to discrete mathematical reasoning."- Arthur Benjamin, Professor of Mathematics at Harvey Mudd College, and author of The Magic of Math

Descriptive set theory has been one of the main areas of research in set theory for almost a century. This text presents a largely balanced approach to the subject, which combines many elements of the different traditions. It includes a wide variety of examples, more than 400 exercises, and applications, in order to illustrate the general concepts and results of the theory.

For a one-semester freshman or sophomore level course on the fundamentals of proof writing or transition to advanced mathematics course. Rather than teach mathematics and the structure of proofs simultaneously, this text first introduces logic as the foundation of proofs and then demonstrates how logic applies to mathematical topics. This method ensures that the students gain a firm understanding of how logic interacts with mathematics and empowers them to solve more complex problems in future math courses.

The Structure of Proof

Set Theory and Its Logic, Revised Edition

A Book of Set Theory

A First Course in Mathematical Logic and Set Theory

Basic Discrete Mathematics