Fractional Differential Equations An Introduction To Fractional Derivatives Fractional Differential Equations To Methods Of Their Solution And Some Mathematics In Science And Engineering

This book gives a practical overview of Fractional Calculus as it relates to Signal Processing

This book is an unique integrated treatise, on the concepts of fractional calculus as models with applications in hydrology, soil science and geomechanics. The models are primarily fractional partial differential equations (fPDEs), and in limited cases, fractional differential equations (fDEs). It develops and applies relevant fPDEs and fDEs mainly to water flow and solute transport in porous media and overland, and in some cases, to concurrent flow and energy transfer. It is an integrated resource with theory and applications for those interested in hydrology, hydraulics and fluid mechanics. The self-contained book summaries the fundamentals for porous media and essential mathematics with extensive references supporting the development of the model and applications.

This book introduces a series of problems and methods insufficiently discussed in the field of Fractional Calculus - a major, emerging tool relevant to all areas of scientific inquiry. The authors present examples based on symbolic computation, written in Maple and Mathematica, and address both mathematical and computational areas in the context of mathematical modeling and the generalization of classical integer-order methods. Distinct from most books, the present volume fills the gap between mathematics and computer fields, and the transition from integer- to fractional-order methods. Introduces Fractional Calculus in an accessible manner, based on standard integer calculus Supports the use of higher-level mathematical packages, such as Mathematica or Maple Facilitates understanding the generalization (towards Fractional Calculus) of important models and systems, such as Lorenz, Chua, and many others Provides a simultaneous introduction to analytical and numerical methods in Fractional Calculus.

In March 2000 leading scientists gathered at the Centro Seminariale Monte Verità, Ascona, Switzerland, for the Third International Symposium on "Fractals 2000 in Biology and Medicine". This interdisciplinary conference provided stimulating contributions from the very topical field Fractals in Biology and Medicine. This volume highlights the growing power and efficacy of the fractal geometry in understanding how to analyze living phenomena and complex shapes.

An Application-Oriented Exposition Using Differential Operators of Caputo Type

An Introduction for Physicists

Theoretical Developments and Applications in Physics and Engineering

The Analysis of Fractional Differential Equations

Functional Fractional Calculus

In this book, there are five chapters: The Laplace Transform, Systems of Homogenous Linear Differential Equations (HLDE), Methods of First and Higher Orders Differential Equations, Extended Methods of First and Higher Orders Differential Equations, and Applications of Differential Equations. In addition, there are exercises at the end of each chapter above to let students practice additional sets of problems other than examples, and they can also check their solutions to some of these exercises by looking at "Answers to Odd-Numbered Exercises" section at the end of this book. This book is a very useful for college students who studied Calculus II, and other students who want to review some concepts of differential equations before studying courses such as partial differential equations, applied mathematics, and electric circuits II.

This graduate textbook provides a self-contained introduction to modern mathematical theory on fractional differential equations. It addresses both ordinary and partial differential equations with a focus on detailed solution theory, especially regularity theory under realistic assumptions on the problem data. The text includes an extensive bibliography, application-driven modeling, extensive exercises, and graphic illustrations throughout to complement its comprehensive presentation of the field. It is recommended for graduate students and researchers in applied and computational mathematics, particularly applied analysis, numerical analysis and inverse problems.

Delay, difference, functional, fractional, and partial differential equations have many applications in science and engineering. In this Special Issue, 29 experts co-authored 10 papers dealing with these subjects. A summary of the main points of these papers follows: Several oscillation conditions for a first-order linear differential equation with non-monotone delay are established in Oscillation Criteria for First Order Differential Equations with Non-Monotone Delays, whereas a sharp oscillation criterion using the notion of slowly varying functions is established in A Sharp Oscillation Criterion for a Linear Differential Equation with Variable Delay. The approximation of a linear autonomous differential equation with a small delay is considered in Approximation of a Linear Autonomous Differential Equation with Small Delay; the model of infection diseases by Marchuk is studied in Around the Model of Infection Disease: The Cauchy Matrix and Its Properties. Exact solutions to fractional-order Fokker–Planck equations are presented in New Exact Solutions and Conservation Laws to the Fractional-Order Fokker–Planck Equations, and a spectral collocation approach to solving a class of time-fractional stochastic heat equations driven by Brownian motion is constructed in A Collocation Approach for Solving Time-Fractional Stochastic Heat Equation Driven by an Additive Noise. A finite difference approximation method for a space fractional convection-diffusion model with variable coefficients is proposed in Finite Difference Approximation Method for a Space Fractional Convection–Diffusion Equation with Variable Coefficients; existence results for a nonlinear fractional difference equation with delay and impulses are established in On Nonlinear Fractional Difference Equation with Delay and Impulses. A complete Noether symmetry analysis of a generalized coupled Lane–Emden–Klein–Gordon–Fock system with central symmetry is provided in Oscillation Criteria for First Order Differential Equations with Non-Monotone Delays, and new soliton solutions of a fractional Jaulent soliton Miodek system via symmetry analysis are presented in New Soliton Solutions of Fractional Jaulent-Miodek System with Symmetry Analysis.

This book introduces a series of problems and methods insufficiently discussed in the field of Fractional Calculus – a major, emerging tool relevant to all areas of scientific inquiry. The authors present examples based on symbolic computation, written in Maple and Mathematica, and address both mathematical and computational areas in the context of mathematical modeling and the generalization of classical integer-order methods. Distinct from most books, the present volume fills the gap between mathematics and computer fields, and the transition from integer- to fractional-order methods.

An Introduction to Fractional Control

Stability, Simulation, Applications and Perspectives

An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their Applications

Basic Theory of Fractional Differential Equations

Fractional Differential Equations, Volume 198: An Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution

This book aims to establish a foundation for fractional derivatives and fractional differential equations. The theory of fractional derivatives enables considering any positive order of differentiation. The history of research in this field is very long, with its origins dating back to Leibniz. Since then, many great mathematicians, such as Abel, have made contributions that cover not only theoretical aspects but also physical applications of fractional calculus. The fractional partial differential equations govern phenomena depending both on spatial and time variables and require more subtle treatments. Moreover, fractional partial differential equations are highly demanded model equations for solving real-world problems such as the anomalous diffusion in heterogeneous media. The studies of fractional partial differential equations have continued to expand explosively. However we observe that available mathematical theory for fractional partial differential equations is not still complete. In particular, operator-theoretical approaches are indispensable for some generalized categories of solutions such as weak solutions, but feasible operator-theoretic foundations for wide applications are not available in monographs. To make this monograph more readable, we are restricting it to a few fundamental types of time-fractional partial differential equations, forgoing many other important and exciting topics such as stability for nonlinear problems. However, we believe that this book works well as an introduction to mathematical research in such vast fields.

In the last two decades, fractional (or non integer) differentiation has played a very important role in various fields such as mechanics, electricity, chemistry, biology, economics, control theory and signal and image processing. For example, in the last three fields, some important considerations such as modelling, curve fitting, filtering, pattern recognition, edge detection, identification, stability, controllability, observability and robustness are now linked to long-range dependence phenomena. Similar progress has been made in other fields listed here. The scope of the book is thus to present the state of the art in the study of fractional systems and the application of fractional differentiation. As this volume covers recent applications of fractional calculus, it will be of interest to engineers, scientists, and applied mathematicians.

Topics in Fractional Differential Equations is devoted to the existence and uniqueness of solutions for various classes of Darboux problems for hyperbolic differential equations or inclusions involving the Caputo fractional derivative. Fractional calculus generalizes the integrals and derivatives to non-integer orders. During the last decade, fractional calculus was found to play a fundamental role in the modeling of a considerable number of phenomena; in particular the modeling of memory-dependent and complex media such as porous media. It has emerged as an important tool for the study of dynamical systems where classical methods reveal strong limitations. Some equations present delays which may be finite, infinite, or state-dependent. Others are subject to an impulsive effect. The above problems are studied using the fixed point approach, the method of upper and lower solution, and the Kuratowski measure of noncompactness. This book is addressed to a wide audience of specialists such as mathematicians, engineers, biologists, and physicists.

Partial differential equations (PDEs) are one of the most used widely forms of mathematics in science and engineering. PDEs can have partial derivatives with respect to (1) an initial value variable, typically time, and (2) boundary value variables, typically spatial variables. Therefore, two fractional PDEs can be considered, (1) fractional in time (TFPDEs), and (2) fractional in space (SFPDEs). The two volumes are directed to the development and use of SFPDEs, with the discussion divided as: Vol 1: Introduction to Algorithms and Computer Coding in R Vol 2: Applications from Classical Integer PDEs. Various definitions of space fractional derivatives have been proposed. We focus on the Caputo derivative, with occasional reference to the Riemann-Liouville derivative. Partial differential equations (PDEs) are one of the most used widely forms of mathematics in science and engineering. PDEs can have partial derivatives with respect to (1) an initial value variable, typically time, and (2) boundary value variables, typically spatial variables. Therefore, two fractional PDEs can be considered, (1) fractional in time (TFPDEs), and (2) fractional in space (SFPDEs). The two volumes are directed to the development and use of SFPDEs, with the discussion divided as: Vol 1: Introduction to Algorithms and Computer Coding in R Vol 2: Applications from Classical Integer PDEs. Various definitions of space fractional derivatives have been proposed. We focus on the Caputo derivative, with occasional reference to the Riemann-Liouville derivative. The Caputo derivative is defined as a convolution integral. Thus, rather than being local (with a value at a particular point in space), the Caputo derivative is non-local (it is based on an integration in space), which is one of the reasons that it has properties not shared by integer derivatives. A principal objective of the two volumes is to provide the reader with a set of documented R routines that are discussed in detail, and can be downloaded and executed without having to first study the details of the relevant numerical analysis and then code a set of routines. In the first volume, the emphasis is on basic concepts of SFPDEs and the associated numerical algorithms. The presentation is not as formal mathematics, e.g., theorems and proofs. Rather, the presentation is by examples of SFPDEs, including a detailed discussion of the algorithms for computing numerical solutions to SFPDEs and a detailed explanation of the associated source code.

Fuzzy Fractional Differential Equations and Applications

A Friendly Introduction to Differential Equations

Fractional Calculus for Scientists and Engineers

Wavelet Methods for Solving Partial Differential Equations and Fractional Differential Equations

Distributed-Order Dynamic Systems

Fractional Differential EquationsAn Introduction to Fractional Derivatives, Fractional Differential Equations, to Methods of Their Solution and Some of Their ApplicationsElsevier

Fractional calculus was first developed by pure mathematicians in the middle of the 19th century. Some 100 years later, engineers and physicists have found applications for these concepts in their areas. However there has traditionally been little interaction between these two communities. In particular, typical mathematical works provide extensive findings on aspects with comparatively little significance in applications, and the engineering literature often lacks mathematical detail and precision. This book bridges the gap between the two communities. It concentrates on the class of fractional derivatives most important in applications, the Caputo operators, and provides a self-contained, thorough and mathematically rigorous study of their properties and of the corresponding differential equations. The text is a useful tool for mathematicians and researchers from the applied sciences alike. It can also be used as a basis for teaching graduate courses on fractional differential equations.

This invaluable monograph is devoted to a rapidly developing area on the research of qualitative theory of fractional ordinary and partial differential equations. It provides the readers the necessary background material required to go further into the subject and explore the rich research literature. The tools used include many classical and modern nonlinear analysis methods such as fixed point theory, measure of noncompactness method, topological degree method, the technique of Picard operators, critical point theory and semigroup theory. Based on the research work carried out by the authors and other experts during the past seven years, the contents are very recent and comprehensive. In this edition, two new topics have been added, that is, fractional impulsive differential equations, and fractional partial differential equations including fractional Navier-Stokes equations and fractional diffusion equations. Contents:Preliminaries:IntroductionSome Notations, Concepts and LemmasFractional CalculusSome Results from Nonlinear AnalysisSemigroupsFractional Functional Differential Equations:IntroductionNeutral Equations with Bounded Delayp-Type Neutral EquationsNeutral Equations with Infinite DelayIterative Functional Differential EquationsNotes and RemarksFractional Ordinary Differential Equations in Banach Spaces:IntroductionCauchy Problems via Measure of Noncompactness MethodCauchy Problems via Topological Degree MethodCauchy Problems via Picard Operators TechniqueNotes and RemarksFractional Abstract Evolution Equations:IntroductionEvolution Equations with Riemann–Liouville DerivativeEvolution Equations with Caputo DerivativeNonlocal Problems for Evolution EquationsAbstract Cauchy Problems with Almost Sectorial OperatorsNotes and RemarksFractional Impulsive Differential Equations:IntroductionImpulsive Initial Value ProblemsImpulsive Boundary Value ProblemsImpulsive Langevin EquationsImpulsive Evolution EquationsNotes and RemarksFractional Boundary Value Problems:IntroductionSolution for BVP with Left and Right Fractional IntegralsMultiple Solutions for BVP with ParametersInfinite Solutions for BVP with Left and Right Fractional IntegralsSolutions for BVP with Left and Right Fractional DerivativesNotes and RemarksFractional Partial Differential Equations:IntroductionFractional Navier–Stokes EquationsFractional Euler–Lagrange EquationsFractional Diffusion EquationsFractional Schrödinger EquationsNotes and Remarks Readership: Researchers and graduate or PhD students dealing with fractional calculus and applied analysis, differential equations and related areas of research.

Commences with the historical development of fractional calculus, its mathematical theory—particularly the Riemann-Liouville version. Numerous examples and theoretical applications of the theory are presented. Features topics associated with fractional differential equations. Discusses Weyl fractional calculus and some of its uses. Includes selected physical problems which lead to fractional differential or integral equations.

Methods of Mathematical Modelling

Fractional Calculus and Its Applications

Fractional Differential Equations: Numerical Methods for Applications

Introduction to the Fractional Calculus of Variations

Theory and Applications of Fractional Differential Equations

Presents a systematic treatment of fuzzy fractional differential equations as well as newly developed computational methods to model uncertain physical problems Complete with comprehensive results and solutions, Fuzzy Arbitrary Order System: Fuzzy Fractional Differential Equations and Applications details newly developed methods of fuzzy computational techniquesneeded to model solve uncertainty. Fuzzy differential equations are solved via various analytical andnumerical methodologies, and this book presents their importance for problem solving, prototypeengineering design, and systems testing in uncertain environments. In recent years, modeling of differential equations for arbitrary and fractional order systems has been increasing in its applicability, and as such, the authors feature examples from a variety of disciplines to illustrate the practicality and importance of the methods within physics, applied mathematics, engineering, and chemistry, to name a few. The fundamentals of fractional differential equations and the basic preliminaries of fuzzy fractional differential equations are first introduced, followed by numerical solutions, comparisons of various methods, and simulated results. In addition, fuzzy ordinary, partial, linear, and nonlinear fractional differential equations are addressed to solve uncertainty in physical systems. In addition, this book features: Basic preliminaries of fuzzy set theory, an introduction of fuzzy arbitrary order differential equations, and various analytical and numerical procedures for solving associated problems Coverage on a variety of fuzzy fractional differential equations including structural, diffusion, and chemical problems as well as heat equations and biomathematical applications Discussions on how to model physical problems in terms of nonprobabilistic methods and provides systematic coverage of fuzzy fractional differential equations and its applications Uncertainties in systems and processes with a fuzzy concept Fuzzy Arbitrary Order System: Fuzzy Fractional Differential Equations and Applications is an ideal resource for practitioners, researchers, and academicians in applied mathematics, physics, biology, engineering, computer science, and chemistry who need to model uncertain physical phenomena and problems. The book is appropriate for graduate-level courses on fractional differential equations for students majoring in applied mathematics, engineering, physics, and computer science.

This book provides a comprehensive set of practical tools for exploring and discovering the world of fractional calculus and its applications, and thereby a means of bridging the theory of fractional differential equations (FDE) with real-world facts. These tools seamlessly blend centuries old numerical methods such as Gaussian quadrature that have stood the test of time with pioneering concepts such as hypermatrix equations to harness the emerging capabilities of modern scientific computing environments. This unique fusion of old and new leads to a unified approach that intuitively parallels the classic theory of differential equations, and results in methods that are unprecedented in computational speed and numerical accuracy. The opening chapter is an introduction to fractional calculus that is geared towards scientists and engineers. The following chapter introduces the reader to the key concepts of approximation theory with an emphasis on the tools of numerical linear algebra. The third chapter provides the keystone for the remainder of the book with a comprehensive set of methods for the approximation of fractional order integrals and derivatives. The fourth chapter describes the numerical solution of initial and boundary value problems for FDE of a single variable, both linear and nonlinear. Moving to two, three, and four dimensions, the ensuing chapter is devoted to a novel approach to the numerical solution of partial FDE that leverages the little-known one-to-one relation between partial differential equations and matrix and hypermatrix equations. The emphasis on applications culminates in the final chapter by addressing inverse problems for ordinary and partial FDE, such as smoothing for data analytics, and the all-important system identification problem. Over a century ago, scientists such as Ludwig Boltzmann and Vito Volterra formulated mathematical models of real materials that -- based on physical evidence -- integrated the history of the system. The present book will be invaluable to students and researchers in fields where analogous phenomena arise, such as viscoelasticity, rheology, polymer dynamics, non-Newtonian fluids, bioengineering, electrochemistry, non-conservative mechanics, groundwater hydrology, NMR and computed tomography, mathematical economics, thermomechanics, anomalous diffusion and transport, control theory, supercapacitors, and genetic algorithms, to name but a few. These investigators will be well-equipped with reproducible numerical methods to explore and discover their particular field of application of FDE.

This book provides a broad overview of the latest developments in fractional calculus and fractional differential equations (FDEs) with an aim to motivate the readers to venture into these areas. It also presents original research describing the fractional operators of variable order, fractional-order delay differential equations, chaos and related phenomena in detail. Selected results on the stability of solutions of nonlinear dynamical systems of the non-commensurate fractional order have also been included. Furthermore, artificial neural network and fractional differential equations are elaborated on; and new transform methods (for example, Sumudu methods) and how they can be employed to solve fractional partial differential equations are discussed. The book covers the latest research on a variety of topics, including: comparison of various numerical methods for solving FDEs, the Adomian decomposition method and its applications to fractional versions of the classical Poisson processes, variable-order fractional operators, fractional variational principles, fractional delay

differential equations, fractional-order dynamical systems and stability analysis, inequalities and comparison theorems in FDEs, artificial neural network approximation for fractional operators, and new transform methods for solving partial FDEs. Given its scope and level of detail, the book will be an invaluable asset for researchers working in these areas.

Fractional calculus provides the possibility of introducing integrals and derivatives of an arbitrary order in the mathematical modelling of physical processes, and it has become a relevant subject with applications to various fields, such as anomalous diffusion, propagation in different media, and propogation in relation to materials with different properties. However, many aspects from theoretical and practical points of view have still to be developed in relation to models based on fractional operators. This Special Issue is related to new developments on different aspects of fractional differential equations, both from a theoretical point of view and in terms of applications in different fields such as physics, chemistry, or control theory, for instance. The topics of the Issue include fractional calculus, the mathematical analysis of the properties of the solutions to fractional equations, the extension of classical approaches, or applications of fractional equations to several fields.

Introduction to Fractional Differential Equations

Fractional Calculus for Hydrology, Soil Science and Geomechanics

Topics in Fractional Differential Equations

Theory, Methods and Applications

Fuzzy Fractional Differential Equations

When a new extraordinary and outstanding theory is stated, it has to face criticism and skeptism, because it is beyond the usual concept. The fractional calculus though not new, was not discussed or developed for a long time, particularly for lack of its application to real life problems. It is extraordinary because it does not deal with "ordinary" differential calculus. It is outstanding because it can now be applied to situations where existing theories fail to give satisfactory results. In this book not only mathematical abstractions are discussed in a lucid manner, with physical mathematical and geometrical explanations, but also several practical applications are given particularly for system identification, description and then efficient controls. The normal physical laws like, transport theory, electrodynamics, equation of motions, elasticity, viscosity, and several others of are based on "ordinary" calculus. In this book these physical laws are generalized in fractional calculus contexts; taking, heterogeneity effect in transport background, the space having traps or islands, irregular distribution of charges, non-ideal spring with mass connected to a pointless-mass ball, material behaving with viscous as well as elastic properties, system relaxation with and without memory, physics of random delay in computer network; and several others; mapping the reality of nature closely. The concept of fractional and complex order differentiation and integration are elaborated mathematically, physically and geometrically with examples. The practical utility of local fractional differentiation for enhancing the character of singularity at phase transition or characterizing the irregularity measure of response function is deliberated. Practical results of viscoelastic experiments, fractional order controls experiments, design of fractional controller and practical circuit synthesis for fractional order elements are elaborated in this book. The book also maps theory of classical integer order differential equations to fractional calculus contexts, and deals in details with conflicting and demanding initialization issues, required in classical techniques. The book presents a modern approach to solve the "solvable" system of fractional and other differential equations, linear, non-linear; without perturbation or transformations, but by applying physical principle of action-and-opposite-reaction, giving "approximately exact" series solutions. Historically, Sir Isaac Newton and Gottfried Wihelm Leibniz independently discovered calculus in the middle of the 17th century. In recognition to this remarkable discovery, J.von Neumann remarked, "...the calculus was the first achievement of modern mathematics and it is difficult to overestimate its importance. I think it defines more equivocally than anything else the inception of modern mathematical analysis which is logical development, still constitute the greatest technical advance in exact thinking." This XXI century has thus started to "think-exactly" for advancement in science & technology by growing application of fractional calculus, and this century has started speaking the language which nature understands the best.

This work aims to present, in a systematic manner, results including the existence and uniqueness of solutions for the Cauchy Type and Cauchy problems involving nonlinear ordinary fractional differential equations.

This is a modified version of Module 10 of the Centre for Mathematical and Statistical Sciences (CMSS). CMSS modules are notes prepared on various topics with many examples from real-life situations and exercises so that the subject matter becomes interesting to students. These modules are used for undergraduate level courses and graduate level training in various topics at CMSS. Aside from Module 8, these modules were developed by Dr A M Mathai, Director of CMSS and Emeritus Professor of Mathematics and Statistics, McGill University, Canada. Module 8 is based on the lecture notes of Professor W J Anderson of McGill University, developed for his undergraduate course (Mathematics 447). Professor Dr Hans J Haubold has been a research collaborator of Dr A M Mathais since 1984, mainly in the areas of astrophysics, special functions and statistical distribution theory. He is also a lifetime member of CMSS and a Professor at CMSS. A large number of papers have been published jointly in these areas since 1984. The following monographs and books have been brought out in conjunction with this joint research: Modern Problems in Nuclear and Neutrino Astrophysics (A M Mathai and H J Haubold, 1988, Akademie-Verlag, Berlin); Special Functions for Applied Scientists (A MMathai and H J Haubold, 2008, Springer, New York); and The H-Function: Theory and Applications (A M Mathai, R K Saxena and H J Haubold, 2010, Springer, New York). These CMSS modules are printed at CMSS Press and published by CMSS. Copies are made available to students free of charge, and to researchers and others at production cost. For the preparation of the initial drafts of all these modules, financial assistance was made available from the Department of Science and Technology, the Government of India (DST), New Delhi under project number SR/S4/MS:287/05. Hence, the authors would like to express their thanks and gratitude to DST, the Government of India, for its financial assistance.

Starting with an introduction to fractional derivatives and numerical approximations, this book presents finite difference methods for fractional differential equations, including time-fractional sub-diffusion equations, time-fractional wave equations, and space-fractional differential equations, among others. Approximation methods for fractional derivatives are developed and approximate accuracies are analyzed in detail.

Finite Difference Methods

Vol 1 - Introduction to Algorithms and Computer Coding in R

Fractional Differential Equations

Time-Fractional Differential Equations

Fractional Calculus and Fractional Differential Equations

This multi-volume handbook is the most up-to-date and comprehensive reference work in the field of fractional calculus and its numerous applications. This second volume collects authoritative chapters covering the mathematical theory of fractional calculus, including ordinary and partial differential equations of fractional order, inverse problems, and evolution equations.

This book presents fractional difference, integral, differential, evolution equations and inclusions, and discusses existence and asymptotic behavior of their solutions. Controllability and relaxed control results are obtained. Combining rigorous deduction with abundant examples, it is of interest to nonlinear science researchers using fractional equations as a tool, and physicists, mechanics researchers and engineers studying relevant topics. Contents Fractional Difference Equations Fractional Integral Equations Fractional Differential Equations Fractional Evolution Equations: Continued Fractional Differential Inclusions

Distributed-order differential equations, a generalization of fractional calculus, are of increasing importance in many fields of science and engineering from the behaviour of complex dielectric media to the modelling of nonlinear systems. This Brief will broaden the toolbox available to researchers interested in modeling, analysis, control and filtering. It contains contextual material outlining the progression from integer-order, through fractional-order to distributed-order systems. Stability issues are addressed with graphical and numerical results highlighting the fundamental differences between constant-, integer-, and distributed-order treatments. The power of the distributed-order model is demonstrated with work on the stability of noncommensurate-order linear time-invariant systems. Generic applications of the distributed-order operator follow: signal processing and viscoelastic damping of a mass–spring set up. A new general approach to discretization of distributed-order derivatives and integrals is described. The Brief is rounded out with a consideration of likely future research and applications and with a number of MATLAB® codes to reduce repetitive coding tasks and encourage new workers in distributed-order systems.

This book is a landmark title in the continuous move from integer to non-integer in mathematics: from integer numbers to real numbers, from factorials to the gamma function, from integer-order models to models of an arbitrary order. For historical reasons, the word 'fractional' is used instead of the word 'arbitrary'. This book is written for readers who are new to the fields of fractional derivatives and fractional-order mathematical models, and feel that they need them for developing more adequate mathematical models. In this book, not only applied scientists, but also pure mathematicians will find fresh motivation for developing new methods and approaches in their fields of research. A reader will find in this book everything necessary for the initial study and immediate application of fractional derivatives fractional differential equations, including several necessary special functions, basic theory of fractional differentiation, uniqueness and existence theorems, analytical numerical methods of solution of fractional differential equations, and many inspiring examples of applications. A unique survey of many applications of fractional calculus Presents basic theory Includes a unified presentation of selected classical results, which are important for applications Provides many examples Contains a separate chapter of fractional order control systems, which opens new perspectives in control theory The first systematic consideration of Caputo's fractional derivative in comparison with other selected approaches Includes tables of fractional derivatives, which can be used for evaluation of all considered types of fractional derivatives

Proceedings of the International Conference held at the University of New Haven, June 1974

An Approach via Fractional Derivatives

Fractional Calculus

An Introduction to the Fractional Calculus and Fractional Differential Equations

Lie Symmetry Analysis of Fractional Differential Equations

Transmutations, Singular and Fractional Differential Equations with Applications to Mathematical Physics connects difficult problems with similar more simple ones. The book's strategy works for differential and integral equations and systems and for many theoretical and applied problems in mathematics, mathematical physics, probability and statistics, applied computer science and numerical methods. In addition to being exposed to recent advances, readers learn to use transmutation methods not only as practical tools, but also as vehicles that deliver theoretical insights. Presents the universal transmutation method as the most powerful for solving many problems in mathematics, mathematical physics, probability and statistics, applied computer science and numerical methods Combines mathematical rigor with an illuminating exposition full of historical notes and fascinating details Enables researchers, lecturers and students to find material under the single "roof"

An Introduction to Fractional Control outlines the theory, techniques and applications of fractional control.

This book contains new and useful materials concerning fuzzy fractional differential and integral operators and their relationship. As the title of the book suggests, the fuzzy subject matter is one of the most important tools discussed. Therefore, it begins by providing a brief but important and new description of fuzzy sets and the computational calculus they require. Fuzzy fractals and fractional operators have a broad range of applications in the engineering, medical and economic sciences. Although these operators have been addressed briefly in previous papers, this book represents the first comprehensive collection of all relevant explanations. Most of the real problems in the biological and engineering sciences involve dynamic models, which are defined by fuzzy fractional operators in the form of fuzzy fractional initial value problems. Another important goal of this book is to solve these systems and analyze their solutions both theoretically and numerically. Given the content covered, the book will benefit all researchers and students in the mathematical and computer sciences, but also the engineering sciences.

The main focus of the book is to implement wavelet based transform methods for solving problems of fractional order partial differential equations arising in modelling real physical phenomena. It explores analytical and numerical approximate solution obtained by wavelet methods for both classical and fractional order partial differential equations.

Fractional-Order Equations and Inclusions

New developments in Functional and Fractional Differential Equations and in Lie Symmetry

Numerical Integration of Space Fractional Partial Differential Equations

An Introduction to Fractional Calculus

Fuzzy Arbitrary Order System

This invaluable book provides a broad introduction to the fascinating and beautiful subject of Fractional Calculus of Variations (FCV). In 1996, FVC evolved in order to better describe non-conservative systems in mechanics. The inclusion of non-conservatism is extremely important from the point of view of applications. Forces that do not store energy are always present in real systems. They remove energy from the systems and, as a consequence, Noether's conservation laws cease to be valid. However, it is still possible to obtain the validity of Noether's principle using FCV. The new theory provides a more realistic approach to physics, allowing us to consider non-conservative systems in a natural way. The authors prove the necessary Euler – Lagrange conditions and corresponding Noether theorems for several types of fractional variational problems, with and without constraints, using Lagrangian and Hamiltonian formalisms. Sufficient optimality conditions are also obtained under convexity, and Leitmann's direct method is discussed within the framework of FCV. The book is self-contained and unified in presentation. It may be used as an advanced textbook by graduate students and ambitious undergraduates in mathematics and mechanics. It provides an opportunity for an introduction to FCV for experienced researchers. The explanations in the book are detailed, in order to capture the interest of the curious reader, and the book provides the necessary background material required to go further into the subject and explore the rich research literature.

Fractional calculus is a collection of relatively little-known mathematical results concerning generalizations of differentiation and integration to noninteger orders. While these results have been accumulated over centuries in various branches of mathematics, they have until recently found little appreciation or application in physics and other mathematically oriented sciences. This situation is beginning to change, and there are now a growing number of research areas in physics which employ fractional calculus. This volume provides an introduction to fractional calculus for physicists, and collects easily accessible review articles surveying those areas of physics in which applications of fractional calculus have recently become prominent. Contents:An Introduction to Fractional Calculus (P L Butzer & U Westphal)Fractional Time Evolution (R Hilfer)Fractional Powers of Infinitesimal Generators of Semigroups (U Westphal)Fractional Differences, Derivatives and Fractal Time Series (B J West & P Grigolini)Fractional Kinetics of Hamiltonian Chaotic Systems (G M Zaslavsky)Polymer Science Applications of Path-Integration, Integral Equations, and Fractional Calculus (J F Douglas)Applications to Problems in Polymer Physics and Rheology (H Schiessel et al.)Applications of Fractional Calculus Techniques to Problems in Biophysics (T F Nonnenmacher & R Metzler)Fractional Calculus and Regular Variation in Thermodynamics (R Hilfer) Readership: Statistical, theoretical and mathematical physicists. Keywords:Fractional Calculus in PhysicsReviews: "This monograph provides a systematic treatment of the theory and applications of fractional calculus for physicists. It contains nine review articles surveying those areas in which fractional calculus has become important. All the chapters are self-contained." Mathematics Abstracts

The trajectory of fractional calculus has undergone several periods of intensive development, both in pure and applied sciences. During the last few decades fractional calculus has also been associated with the power law effects and its various applications. It is a natural to ask if fractional calculus, as a nonlocal calculus, can produce new results within the well-established field of Lie symmetries and their applications. In Lie Symmetry Analysis of Fractional Differential Equations the authors try to answer this vital question by analyzing different aspects of fractional Lie symmetries and related conservation laws. Finding the exact solutions of a given fractional partial differential equation is not an easy task, but is one that the authors seek to grapple with here. The book also includes generalization of Lie symmetries for fractional integro differential equations. Features Provides a solid basis for understanding fractional calculus, before going on to explore in detail Lie Symmetries and their applications Useful for PhD and postdoc graduates, as well as for all mathematicians and applied researchers who use the powerful concept of Lie symmetries Filled with various examples to aid understanding of the topics

The book presents a concise introduction to the basic methods and strategies in fractional calculus which enables the reader to catch up with the state-of-the-art in this field and to participate and contribute in the development of this exciting research area. This book is devoted to the application of fractional calculus on physical problems. The fractional concept is applied to subjects in classical mechanics, image processing, folded potentials in cluster physics, infrared spectroscopy, group theory, quantum mechanics, nuclear physics, hadron spectroscopy up to quantum field theory and will surprise the reader with new intriguing insights. This new, extended edition includes additional chapters about numerical solution of the fractional Schr ö dinger equation, self-similarity and the geometric interpretation of non-isotropic fractional differential operators. Motivated by the positive response, new exercises with elaborated solutions are added, which significantly support a deeper understanding of the general aspects of the theory. Besides students as well as researchers in this field, this book will also be useful as a supporting medium for teachers teaching courses devoted to this subject.

Applications of Fractional Calculus in Physics

A Theoretical Introduction

Advances in Fractional Calculus

Fuzzy Fractional Differential Operators and Equations

This book features original research articles on the topic of mathematical modelling and fractional differential equations. The contributions, written by leading researchers in the field, consist of chapters on classical and modern dynamical systems modelled by fractional differential equations in physics, engineering, signal processing, fluid mechanics, and bioengineering, manufacturing, systems engineering, and project management. The book offers theory and practical applications for the solutions of real-life problems and will be of interest to graduate level students, educators, researchers, and scientists interested in mathematical modelling and its diverse applications. Features Presents several recent developments in the theory and applications of fractional calculus Includes chapters on different analytical and numerical methods dedicated to several mathematical equations Develops methods for the mathematical models which are governed by fractional differential equations Provides methods for models in physics, engineering, signal processing, fluid mechanics, and bioengineering Discusses real-world problems, theory, and applications

FRACTIONAL CALCULUS: Theory and Applications deals with differentiation and integration of arbitrary order. The origin of this subject can be traced back to the end of seventeenth century, the time when Newton and Leibniz developed foundations of differential and integral calculus. Nonetheless, utility and applicability of FC to various branches of science and engineering have been realized only in last few decades. Recent years have witnessed tremendous upsurge in research activities related to the applications of FC in modeling of real-world systems. Unlike the derivatives of integral order, the non-local nature of fractional derivatives correctly models many natural phenomena containing long memory and give more accurate description than their integer counterparts.The present book comprises of contributions from academicians and leading researchers and gives a panoramic overview of various aspects of this subject: Introduction to Fractional Calculus Fractional Differential Equations Fractional Ordered Dynamical Systems Fractional Operators on Fractals Local Fractional Derivatives Fractional Control Systems Fractional Operators and Statistical Distributions Applications to Engineering

This invaluable book provides a broad introduction to the fascinating and beautiful subject of Fractional Calculus of Variations (FCV). In 1996, FVC evolved in order to better describe non-conservative systems in mechanics. The inclusion of non-conservatism is extremely important from the point of view of applications. Forces that do not store energy are always present in real systems. They remove energy from the systems and, as a consequence, Noether's conservation laws cease to be valid. However, it is still possible to obtain the validity of Noether's principle using FCV. The new theory provides a more realistic approach to physics, allowing us to consider non-conservative systems in a natural way. The authors prove the necessary Euler Lagrange conditions and corresponding Noether theorems for several types of fractional variational problems, with and without constraints, using Lagrangian and Hamiltonian formalisms. Sufficient optimality conditions are also obtained under convexity, and Leitmann's direct method is discussed within the framework of FCV. The book is self-contained and unified in presentation. It may be used as an advanced textbook by graduate students and ambitious undergraduates in mathematics and mechanics. It provides an opportunity for an introduction to FCV for experienced researchers. The explanations in the book are detailed, in order to capture the interest of the curious reader, and the book provides the necessary background material required to go further into the subject and explore the rich research literature.

Fractals in Biology and Medicine

Transmutations, Singular and Fractional Differential Equations with Applications to Mathematical Physics

An Introduction to Applications