Convex Optimization Theory Chapter 2 Exercises And

This systematic and comprehensive account of asymptotic sets and functions develops a broad and useful theory in the areas of optimization and variational inequalities. The central focus is on problems of handling unbounded situations, using solutions of a given problem in these classes, when for example standard compacity hypothesis is not present. This book will interest advanced graduate students, researchers, and practitioners of optimization theory, nonlinear programming, and applied mathematics.

This textbook offers graduate students a concise introduction to the classic notions of convex optimization. Written in a highly accessible style and including numerous examples and illustrations, it presents everything readers need to know about convexity and convex optimization. The book introduces a systematic three-step method for doing everything, which can be summarized as "conify, work, deconify". It starts with the concept of convex sets, their primal description, constructions, topological properties and dual description, and then moves on to convex functions and the fundamental principles of convex optimization and their use in the complete analysis of convex optimization problems by means of a systematic four-step method. Lastly, it includes

chapters on alternative formulations of optimality conditions and on illustrations of their use. "The author deals with the delicate subjects in a precise yet light-minded spirit... For experts in the field, this book not only offers a unifying view, but also opens a door to new discoveries in convexity and optimization...perfectly suited for classroom teaching." Shuzhong Zhang, Professor of Industrial and Systems Engineering, University of Minnesota

An insightful, concise, and rigorous treatment of the basic theory of convex sets and functions in finite dimensions, and the analytical/geometrical foundations of convex optimization and duality theory. Convexity theory is first developed in a simple accessible manner, using easily visualized proofs. Then the focus shifts to a transparent geometrical line of analysis to develop the fundamental duality between descriptions of convex functions in terms of points, and in terms of hyperplanes. Finally, convexity theory and abstract duality are applied to problems of constrained optimization, Fenchel and conic duality, and game theory to develop the sharpest possible duality results within a highly visual geometric framework. This on-line version of the book, includes an extensive set of theoretical problems with detailed high-quality solutions, which significantly extend the range and value of the book. The book may be used as a text for a theoretical convex optimization course; the author has taught

several variants of such a course at MIT and elsewhere over the last ten years. It may also be used as a supplementary source for nonlinear programming classes, and as a theoretical foundation for classes focused on convex optimization models (rather than theory). It is an excellent supplement to several of our books: Convex Optimization Algorithms (Athena Scientific, 2015), Nonlinear Programming (Athena Scientific, 2017), Network Optimization(Athena Scientific, 1998), Introduction to Linear Optimization (Athena Scientific, 1997), and Network Flows and Monotropic Optimization (Athena Scientific, 1998).

Although LMI has emerged as a powerful tool with applications across the major domains of systems and control, there has been a need for a textbook that provides an accessible introduction to LMIs in control systems analysis and design. Filling this need, LMIs in Control Systems: Analysis, Design and Applications focuses on the basic analysis and d

Code Generation for Embedded Convex Optimization

Quasiconvex Optimization and Location Theory

Polyhedral and Semidefinite Programming Methods in Combinatorial Optimization

Convex Optimization

Convex Optimization with Computational Errors

Optimality Conditions in Convex Optimization

This book constitutes the proceedings of the 26th International Conference on

Algorithmic Learning Theory, ALT 2015, held in Banff, AB, Canada, in October 2015, and co-located with the 18th International Conference on Discovery Science, DS 2015. The 23 full papers presented in this volume were carefully reviewed and selected from 44 submissions. In addition the book contains 2 full papers summarizing the invited talks and 2 abstracts of invited talks. The papers are organized in topical sections named: inductive inference; learning from queries, teaching complexity; computational learning theory and algorithms; statistical learning theory and sample complexity; online learning, stochastic optimization; and Kolmogorov complexity, algorithmic information theory.

Optimality Conditions in Convex Optimization explores an important and central issue in the field of convex optimization: optimality conditions. It brings together the most important and recent results in this area that have been scattered in the literature—notably in the area of convex analysis—essential in developing many of the important results in this book, and not usually found in conventional texts. Unlike other books on convex optimization, which usually discuss

algorithms along with some basic theory, the sole focus of this book is on fundamental and advanced convex optimization theory. Although many results presented in the book can also be proved in infinite dimensions, the authors focus on finite dimensions to allow for much deeper results and a better understanding of the structures involved in a convex optimization problem. They address semi-infinite optimization problems; approximate solution concepts of convex optimization problems; and some classes of non-convex problems which can be studied using the tools of convex analysis. They include examples wherever needed, provide details of major results, and discuss proofs of the main results.
New edition of a graduate-level textbook on that focuses on online convex optimization, a machine learning framework that views optimization as a process. In many practical applications, the environment is so complex that it is not feasible to lay out a comprehensive theoretical model and use classical algorithmic theory and/or mathematical optimization. Introduction to Online Convex Optimization presents a robust machine learning approach that contains elements of mathematical optimization,

game theory, and learning theory: an optimization method that learns from experience as more aspects of the problem are observed. This view of optimization as a process has led to some spectacular successes in modeling and systems that have become part of our daily lives. Based on the "Theoretical Machine Learning" course taught by the author at Princeton University, the second edition of this widely used graduate level text features: Thoroughly updated material throughout New chapters on boosting, adaptive regret, and approachability and expanded exposition on optimization Examples of applications, including prediction from expert advice, portfolio selection, matrix completion and recommendation systems, SVM training, offered throughout Exercises that guide students in completing parts of proofs

A comprehensive introduction to the tools, techniques and applications of convex optimization.

Theory, Algorithms, and Applications with MATLAB

Applied Nonlinear Analysis

A Finite-Dimensional View

Algorithms and Complexity

Introduction to Online Convex Optimization, second edition

Convex Analysis and Optimization

This book serves as a reference for a self-contained course on online convex optimization and the convex optimization approach to machine learning for the educated graduate student in computer science/electrical engineering/ operations research/statistics and related fields. An ideal reference.

Convex optimization is widely used, in many fields, but is nearly always constrained to problems solved in a few minutes or seconds, and even then, nearly always with a human in the loop. The advent of parser-solvers has made convex optimization simpler and more accessible, and greatly increased the number of people using convex optimization. Most current applications, however, are for the design of systems or analysis of data. It is possible to use convex optimization for real-time or embedded applications, where the optimization solver is a part of a larger system. Here, the optimization algorithm must find solutions much faster than a generic solver, and often has a hard, real-time deadline. Use in embedded applications additionally means that the solver cannot fail, and must be robust even in the presence of relatively poor quality data. For ease of embedding, the solver should be simple, and have minimal dependencies on external libraries. Convex optimization has been successfully applied in such settings in the past. However, they have usually necessitated a custom, hand-written solver. This requires

signficant time and expertise, and has been a major factor preventing the adoption of convex optimization in embedded applications. This work describes the implementation and use of a prototype code generator for convex optimization, CVXGEN, that creates high-speed solvers automatically. Using the principles of disciplined convex programming, CVXGEN allows the user to describe an optimization problem in a convenient, high-level language, then receive code for compilation into an extremely fast, robust, embeddable solver.
This work is intended to serve as a guide for graduate students and researchers who wish to get acquainted with the main theoretical and practical tools for the numerical minimization of convex functions on Hilbert spaces. Therefore, it contains the main tools that are necessary to conduct independent research on the topic. It is also a concise, easy-to-follow and self-contained textbook, which may be useful for any researcher working on related fields, as well as teachers giving graduate-level courses on the topic. It will contain a thorough revision of the extant literature including both classical and state-of-the-art references.
Mathematical Models is a component of Encyclopedia of Mathematical Sciences in the global Encyclopedia of Life Support Systems (EOLSS), which is an integrated compendium of twenty one Encyclopedias. The Theme on Mathematical Models discusses matters of great relevance to our world such as: Basic

Principles of Mathematical Modeling; Mathematical Models in Water Sciences; Mathematical Models in Energy Sciences; Mathematical Models of Climate and Global Change; Infiltration and Ponding; Mathematical Models of Biology; Mathematical Models in Medicine and Public Health; Mathematical Models of Society and Development. These three volumes are aimed at the following five major target audiences: University and College students Educators, Professional practitioners, Research personnel and Policy analysts, managers, and decision makers and NGOs.

Convex Analysis and Nonlinear Optimization
Semidefinite Optimization and Convex Algebraic Geometry
Optimization for Machine Learning
Convex Optimization Algorithms
Optimization—Theory and Practice
Interior-point Polynomial Algorithms in Convex Programming

This reference text, now in its second edition, offers a modern unifying presentation of three basic areas of nonlinear analysis: convex analysis, monotone operator theory, and the fixed point theory of nonexpansive operators. Taking a unique comprehensive approach, the theory is developed from the ground up, with the rich connections and interactions between the areas as the central focus, and it is illustrated by a large number of examples. The Hilbert space setting of the material offers a wide range of

applications while avoiding the technical difficulties of general Banach spaces. The authors have also drawn upon recent advances and modern tools to simplify the proofs of key results making the book more accessible to a broader range of scholars and users. Combining a strong emphasis on applications with exceptionally lucid writing and an abundance of exercises, this text is of great value to a large audience including pure and applied mathematicians as well as researchers in engineering, data science, machine learning, physics, decision sciences, economics, and inverse problems. The second edition of Convex Analysis and Monotone Operator Theory in Hilbert Spaces greatly expands on the first edition, containing over 140 pages of new material, over 270 new results, and more than 100 new exercises. It features a new chapter on proximity operators including two sections on proximity operators of matrix functions, in addition to several new sections distributed throughout the original chapters. Many existing results have been improved, and the list of references has been updated. Heinz H. Bauschke is a Full Professor of Mathematics at the Kelowna campus of the University of British Columbia, Canada. Patrick L. Combettes, IEEE Fellow, was on the faculty of the City University of New York and of Université Pierre et Marie Curie – Paris 6 before joining North Carolina State University as

a Distinguished Professor of Mathematics in 2016.

Specialists working in the areas of optimization, mathematical programming, or control theory will find this book invaluable for studying interior-point methods for linear and quadratic programming, polynomial-time methods for nonlinear convex programming, and efficient computational methods for control problems and variational inequalities. A background in linear algebra and mathematical programming is necessary to understand the book. The detailed proofs and lack of "numerical examples" might suggest that the book is of limited value to the reader interested in the practical aspects of convex optimization, but nothing could be further from the truth. An entire chapter is devoted to potential reduction methods precisely because of their great efficiency in practice.

It was in the middle of the 1980s, when the seminal paper by Kar markar opened a new epoch in nonlinear optimization. The importance of this paper, containing a new polynomial-time algorithm for linear op timization problems, was not only in its complexity bound. At that time, the most surprising feature of this algorithm was that the theoretical pre diction of its high efficiency was supported by excellent computational results. This unusual fact dramatically changed the style and direc tions of the research in nonlinear optimization.

Thereafter it became more and more common that the new methods were provided with a complexity analysis, which was considered a better justification of their efficiency than computational experiments. In a new rapidly develop ing field, which got the name "polynomial-time interior-point methods", such a justification was obligatory. Afteralmost fifteen years of intensive research, the main results of this development started to appear in monographs [12, 14, 16, 17, 18, 19]. Approximately at that time the author was asked to prepare a new course on nonlinear optimization for graduate students. The idea was to create a course which would reflect the new developments in the field. Actually, this was a major challenge. At the time only the theory of interior-point methods for linear optimization was polished enough to be explained to students. The general theory of self-concordant functions had appeared in print only once in the form of research monograph [12].

This book presents the mathematical theory of vector variational inequalities and their relations with vector optimization problems. It is the first-ever book to introduce well-posedness and sensitivity analysis for vector equilibrium problems. The first chapter provides basic notations and results from the areas of convex analysis, functional analysis, set-valued analysis and fixed-point theory for set-valued maps, as

well as a brief introduction to variational inequalities and equilibrium problems. Chapter 2 presents an overview of analysis over cones, including continuity and convexity of vector-valued functions. The book then shifts its focus to solution concepts and classical methods in vector optimization. It describes the formulation of vector variational inequalities and their applications to vector optimization, followed by separate chapters on linear scalarization, nonsmooth and generalized vector variational inequalities. Lastly, the book introduces readers to vector equilibrium problems and generalized vector equilibrium problems. Written in an illustrative and reader-friendly way, the book offers a valuable resource for all researchers whose work involves optimization and vector optimization.

MATHEMATICAL MODELS – Volume III

Convex Analysis and Monotone Operator Theory in Hilbert Spaces

Conjugate Duality in Convex Optimization

Algorithms for Convex Optimization

Convexity and Optimization in Finite Dimensions I

Introduction to Online Convex Optimization

grams of which the objective is given by the ratio of a convex by a positive (over a convex domain) concave function. As observed by Sniedovich (Ref. [102, 103]) most of the properties of fractional pro grams could be found in other programs, given that the objective function could be written as a particular composition of functions. He called this new field C programming, standing for composite concave programming. In his seminal book on dynamic programming (Ref. [104]), Sniedovich shows how the study of such com positions can help tackling non-separable dynamic programs that otherwise would defeat solution. Barros and Frenk (Ref. [9]) developed a cutting plane algorithm capable of optimizing C-programs. More recently, this algorithm has been used by Carrizosa and Plastria to solve a global optimization problem in facility location (Ref. [16]). The distinction between global optimization problems (Ref. [54]) and generalized convex problems can sometimes be hard to establish. That is exactly the reason why so much effort has been placed into finding an exhaustive classification of the different weak forms of convexity, establishing a new definition just to satisfy some desirable property in the most general way possible. This book does not aim at all the subtleties of the different generalizations of convexity, but concentrates on the most general of them all, quasiconvex programming. Chapter 5 shows clearly where the real difficulties appear. Dantzig's development of linear programming into one of the most applicable optimization techniques has spread interest in the algebra of linear inequalities, the geometry of polyhedra, the topology of convex sets, and the analysis of convex functions. It is the goal of this volume to provide a synopsis of these topics, and thereby the theoretical back ground for the arithmetic of convex optimization to be treated in a sub sequent volume. The exposition of each chapter is essentially independent, and attempts to reflect a specific style of mathematical reasoning. The emphasis lies on linear and convex duality theory, as initiated by Gale, Kuhn and Tucker, Fenchel, and v. Neumann, because it represents the theoretical development whose impact on modern optimi zation techniques has been the most pronounced. Chapters 5 and 6 are devoted to two characteristic aspects of duality theory: conjugate functions or polarity on the one hand, and saddle points on the other. The Farkas lemma on linear inequalities and its generalizations, Motzkin's description of polyhedra, Minkowski's supporting plane theorem are indispensable elementary tools which are contained in chapters 1, 2 and 3, respectively. The treatment of extremal properties of polyhedra as well as of general convex sets is based on the far reaching work of Klee. Chapter 2 terminates with a description of Gale diagrams, a recently developed successful technique for exploring polyhedral structures.

This monograph presents the main complexity theorems in convex optimization and their corresponding algorithms. It begins with the fundamental theory of black-box optimization and proceeds to guide the reader through recent advances in structural optimization and stochastic optimization. The presentation of black-box optimization, strongly influenced by the seminal book by Nesterov, includes the analysis of cutting plane methods, as well as (accelerated) gradient descent schemes. Special attention is also given to non-Euclidean settings (relevant algorithms include Frank-Wolfe, mirror descent, and dual averaging), and discussing their relevance in machine learning. The text provides a gentle introduction to structural optimization with FISTA (to optimize a sum of a smooth and a simple non-smooth term), saddle-point mirror prox (Nemirovski's alternative to Nesterov's smoothing), and a concise description of interior point methods. In stochastic optimization it discusses stochastic gradient descent, mini-batches, random coordinate descent, and sublinear algorithms. It also briefly touches upon convex relaxation of combinatorial problems and the use of randomness to round solutions, as well as random walks based methods.

Convex optimization has an increasing impact on many areas of mathematics, applied sciences, and practical applications. It is now being taught at many universities and being used by researchers of different fields. As convex analysis is the mathematical f

Lectures on Convex Optimization
Analysis, Design and Applications
Theory and Examples
Algorithmic Learning Theory
Convex Optimization in Normed Spaces
Theory, Methods and Examples

In the last few years, Algorithms for Convex Optimization have revolutionized algorithm design, both for discrete and continuous optimization problems. For problems like maximum flow, maximum matching, and submodular function minimization, the fastest algorithms involve essential methods such as gradient descent, mirror descent, interior point methods, and ellipsoid methods. The goal of this self-contained book is to enable researchers and professionals in computer science, data science, and machine learning to gain an in-depth understanding of these algorithms. The text emphasizes how to derive key algorithms for convex optimization from first principles and how to establish precise running time bounds. This modern text explains the success of these algorithms in problems of discrete optimization, as well as how these methods have significantly pushed the state of the art of convex optimization itself.

"This book concerns matter that is intrinsically difficult: convex optimization, complementarity and duality, nonsmooth analysis, linear and

nonlinear programming, etc. The author has skillfully introduced these and many more concepts, and woven them into a seamless whole by retaining an easy and consistent style throughout. The book is not all theory: There are many real-life applications in structural engineering, cable networks, frictional contact problems, and plasticity... I recommend it to any reader who desires a modern, authoritative account of nonsmooth mechanics and convex optimization." — Prof. Graham M.L. Gladwell, Distinguished Professor Emeritus, University of Waterloo, Fellow of the Royal Society of Canada "... reads very well—the structure is good, the language and style are clear and fluent, and the material is rendered accessible by a careful presentation that contains many concrete examples. The range of applications, particularly to problems in mechanics, is admirable and a valuable complement to theoretical and computational investigations that are at the forefront of the areas concerned." — Prof. B. Daya Reddy, Department of Mathematics and Applied Mathematics, Director of Centre for Research in Computational and Applied Mechanics, University of Cape Town, South Africa "Many materials and structures (e.g., cable networks, membrane) involved in practical engineering applications have complex responses that

cannot be described by smooth constitutive relations. ... The author shows how these difficult problems can be tackled in the framework of convex analysis by arranging the carefully chosen materials in an elegant way. Most of the contents of the book are from the original contributions of the author. They are both mathematically rigorous and readable. This book is a must-read for anyone who intends to get an authoritative and state-of-art description for the analysis of nonsmooth mechanics problems with theory and tools from convex analysis." — Prof. Xu Guo, State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology

The book is devoted to the study of approximate solutions of optimization problems in the presence of computational errors. It contains a number of results on the convergence behavior of algorithms in a Hilbert space, which are known as important tools for solving optimization problems. The research presented in the book is the continuation and the further development of the author's (c) 2016 book Numerical Optimization with Computational Errors, Springer 2016. Both books study the algorithms taking into account computational errors which are always present in practice. The

main goal is, for a known computational error, to find out what an approximate solution can be obtained and how many iterates one needs for this. The main difference between this new book and the 2016 book is that in this present book the discussion takes into consideration the fact that for every algorithm, its iteration consists of several steps and that computational errors for different steps are generally, different. This fact, which was not taken into account in the previous book, is indeed important in practice. For example, the subgradient projection algorithm consists of two steps. The first step is a calculation of a subgradient of the objective function while in the second one we calculate a projection on the feasible set. In each of these two steps there is a computational error and these two computational errors are different in general. It may happen that the feasible set is simple and the objective function is complicated. As a result, the computational error, made when one calculates the projection, is essentially smaller than the computational error of the calculation of the subgradient. Clearly, an opposite case is possible too. Another feature of this book is a study of a number of important algorithms which appeared recently in the literature and which are not discussed in the previous book. This monograph contains 12

chapters. Chapter 1 is an introduction. In Chapter 2 we study the subgradient projection algorithm for minimization of convex and nonsmooth functions. We generalize the results of [NOCE] and establish results which has no prototype in [NOCE]. In Chapter 3 we analyze the mirror descent algorithm for minimization of convex and nonsmooth functions, under the presence of computational errors. For this algorithm each iteration consists of two steps. The first step is a calculation of a subgradient of the objective function while in the second one we solve an auxiliary minimization problem on the set of feasible points. In each of these two steps there is a computational error. We generalize the results of [NOCE] and establish results which has no prototype in [NOCE]. In Chapter 4 we analyze the projected gradient algorithm with a smooth objective function under the presence of computational errors. In Chapter 5 we consider an algorithm, which is an extension of the projection gradient algorithm used for solving linear inverse problems arising in signal/image processing. In Chapter 6 we study continuous subgradient method and continuous subgradient projection algorithm for minimization of convex nonsmooth functions and for computing the saddle points of convex-concave functions, under the presence of

computational errors. All the results of this chapter has no prototype in [NOCE]. In Chapters 7-12 we analyze several algorithms under the presence of computational errors which were not considered in [NOCE]. Again, each step of an iteration has a computational errors and we take into account that these errors are, in general, different. An optimization problems with a composite objective function is studied in Chapter 7. A zero-sum game with two-players is considered in Chapter 8. A predicted decrease approximation-based method is used in Chapter 9 for constrained convex optimization. Chapter 10 is devoted to minimization of quasiconvex functions. Minimization of sharp weakly convex functions is discussed in Chapter 11. Chapter 12 is devoted to a generalized projected subgradient method for minimization of a convex function over a set which is not necessarily convex. The book is of interest for researchers and engineers working in optimization. It also can be useful in preparation courses for graduate students. The main feature of the book which appeals specifically to this audience is the study of the influence of computational errors for several important optimization algorithms. The book is of interest for experts in applications of optimization to engineering and economics.

This book provides a comprehensive and

This book provides a comprehensive and

accessible presentation of algorithms for solving convex optimization problems. It relies on rigorous mathematical analysis, but also aims at an intuitive exposition that makes use of visualization where possible. This is facilitated by the extensive use of analytical and algorithmic concepts of duality, which by nature lend themselves to geometrical interpretation. The book places particular emphasis on modern developments, and their widespread applications in fields such as large-scale resource allocation problems, signal processing, and machine learning. The book is aimed at students, researchers, and practitioners, roughly at the first year graduate level. It is similar in style to the author's 2009"Convex Optimization Theory" book, but can be read independently. The latter book focuses on convexity theory and optimization duality, while the present book focuses on algorithmic issues. The two books share notation, and together cover the entire finite-dimensional convex optimization methodology. To facilitate readability, the statements of definitions and results of the "theory book" are reproduced without proofs in Appendix B.

Convex Analysis for Optimization

Utility-Based Learning from Data

A Basic Course

Convex Optimization for Signal Processing and Communications
Lectures on Modern Convex Optimization
Introductory Lectures on Convex Optimization

This book provides a comprehensive, modern introduction to convex optimization, a field that is becoming increasingly important in applied mathematics, economics and finance, engineering, and computer science, notably in data science and machine learning. Written by a leading expert in the field, this book includes recent advances in the algorithmic theory of convex optimization, naturally complementing the existing literature. It contains a unified and rigorous presentation of the acceleration techniques for minimization schemes of first- and second-order. It provides readers with a full treatment of the smoothing technique, which has tremendously extended the abilities of gradient-type methods. Several powerful approaches in structural optimization, including optimization in relative scale and polynomial-time interior-point methods, are also discussed in detail. Researchers in theoretical optimization as well as professionals working on optimization problems will find this book very useful. It presents many successful examples of how to develop very fast specialized minimization algorithms. Based on the author's lectures, it can naturally serve as the basis for introductory and advanced courses in convex optimization for students in engineering, economics, computer science and mathematics.

Convex Optimization for Signal Processing and Communications: From Fundamentals to Applications provides fundamental background knowledge of convex optimization, while striking a balance between mathematical theory and applications in signal processing and communications. In addition to comprehensive proofs and perspective interpretations for core convex optimization theory, this book

also provides many insightful figures, remarks, illustrative examples, and guided journeys from theory to cutting-edge research explorations, for efficient and in-depth learning, especially for engineering students and professionals. With the powerful convex optimization theory and tools, this book provides you with a new degree of freedom and the capability of solving challenging real-world scientific and engineering problems.

A uniquely pedagogical, insightful, and rigorous treatment of the analytical/geometrical foundations of optimization. The book provides a comprehensive development of convexity theory, and its rich applications in optimization, including duality, minimax/saddle point theory, Lagrange multipliers, and Lagrangian relaxation/nondifferentiable optimization. It is an excellent supplement to several of our books: Convex Optimization Theory (Athena Scientific, 2009), Convex Optimization Algorithms (Athena Scientific, 2015), Nonlinear Programming (Athena Scientific, 2016), Network Optimization (Athena Scientific, 1998), and Introduction to Linear Optimization (Athena Scientific, 1997). Aside from a thorough account of convex analysis and optimization, the book aims to restructure the theory of the subject, by introducing several novel unifying lines of analysis, including: 1) A unified development of minimax theory and constrained optimization duality as special cases of duality between two simple geometrical problems. 2) A unified development of conditions for existence of solutions of convex optimization problems, conditions for the minimax equality to hold, and conditions for the absence of a duality gap in constrained optimization. 3) A unification of the major constraint qualifications allowing the use of Lagrange multipliers for nonconvex constrained optimization, using the notion of constraint pseudonormality and an enhanced form of the Fritz John necessary optimality conditions. Among its features the book: a) Develops rigorously

and comprehensively the theory of convex sets and functions, in the classical tradition of Fenchel and Rockafellar b) Provides a geometric, highly visual treatment of convex and nonconvex optimization problems, including existence of solutions, optimality conditions, Lagrange multipliers, and duality c) Includes an insightful and comprehensive presentation of minimax theory and zero sum games, and its connection with duality d) Describes dual optimization, the associated computational methods, including the novel incremental subgradient methods, and applications in linear, quadratic, and integer programming e) Contains many examples, illustrations, and exercises with complete solutions (about 200 pages) posted at the publisher's web site http://www.athenasc.com/convexity.html

Utility-Based Learning from Data provides a pedagogical, self-contained discussion of probability estimation methods via a coherent approach from the viewpoint of a decision maker who acts in an uncertain environment. This approach is motivated by the idea that probabilistic models are usually not learned for their own sake; rather, they are used t

Introduction to Nonlinear Optimization

26th International Conference, ALT 2015, Banff, AB, Canada, October 4-6, 2015, Proceedings

LMIs in Control Systems

Nonsmooth Mechanics and Convex Optimization

A Unified Approach

Optimization is a field important in its own right but is also integral to numerous applied sciences, including operations research, management science, economics, finance and all branches of mathematics-oriented engineering. Constrained optimization models are one of the most widely used mathematical models in operations research and management science. This book gives a modern and well-balanced presentation of the subject, focusing on theory but also including algorithims and examples from various real-world applications. Detailed examples and counter-examples are provided--as are exercises, solutions and helpful hints, and Matlab/Maple supplements.

Optimization is a rich and thriving mathematical discipline, and the underlying theory of current computational optimization techniques grows ever more sophisticated. This book aims to provide a concise, accessible account of convex analysis and its applications and extensions, for a broad audience. Each section concludes with an often extensive set of optional exercises. This new edition adds material on semismooth optimization, as well as several new proofs.

An up-to-date account of the interplay between optimization and machine learning, accessible to students and researchers in both communities. The interplay between optimization and machine learning is one of the most important developments in modern computational science. Optimization formulations and methods are proving to be vital in designing algorithms to extract essential knowledge from huge volumes of data. Machine learning, however, is not simply a consumer of optimization technology but a rapidly evolving field that is itself generating new optimization ideas. This book captures the state of the art of the interaction between optimization and machine learning in a way that is accessible to researchers in both fields. Optimization approaches have enjoyed prominence in machine learning because of their wide applicability and attractive theoretical properties. The increasing complexity, size, and variety of today's machine learning models call for the reassessment of existing assumptions. This book starts the process of reassessment. It describes the resurgence in novel contexts of established frameworks such as first-order methods, stochastic approximations, convex relaxations, interior-point methods, and proximal methods. It also devotes attention to newer themes such as regularized optimization, robust optimization, gradient and subgradient methods, splitting techniques, and second-order methods. Many of these techniques draw inspiration from other fields, including operations research, theoretical computer science, and subfields of optimization. The book will enrich the ongoing cross-fertilization between the machine learning community and these other fields, and within the broader optimization community.

Since the early 1960s, polyhedral methods have played a central role in both the theory and practice of combinatorial optimization. Since the early 1990s, a new technique, semidefinite programming, has been increasingly applied to some combinatorial optimization problems. The semidefinite programming problem is the problem of optimizing a linear function of matrix variables, subject to finitely many linear inequalities and the positive semidefiniteness condition on some of the matrix variables. On certain problems, such as maximum cut, maximum satisfiability, maximum stable set and geometric representations of graphs, semidefinite programming techniques yield important new results. This monograph provides the necessary background to work with semidefinite optimization techniques, usually by drawing parallels to the development of polyhedral techniques and with a special focus on combinatorial optimization, graph theory and lift-and-project methods. It allows the reader to rigorously develop the necessary knowledge, tools and skills to work in the area that is at the intersection of combinatorial optimization and semidefinite optimization. A solid background in mathematics at the undergraduate level and some exposure to linear optimization are required. Some familiarity with computational complexity theory and the analysis of algorithms would be helpful. Readers with these prerequisites will appreciate the important open problems and exciting new directions as well as new connections to other areas in mathematical sciences that the book provides.

An Easy Path to Convex Analysis and Applications

Analysis, Algorithms, and Engineering Applications
Convex Analysis and Global Optimization
Convex Optimization & Euclidean Distance Geometry
From Fundamentals to Applications
Asymptotic Cones and Functions in Optimization and Variational Inequalities

The study of Euclidean distance matrices (EDMs) fundamentally asks what can be known geometrically given onlydistance information between points in Euclidean space. Each point may represent simply locationor, abstractly, any entity expressible as a vector in finite-dimensional Euclidean space.The answer to the question posed is that very much can be known about the points;the mathematics of this combined study of geometry and optimization is rich and deep.Throughout we cite beacons of historical accomplishment.The application of EDMs has already proven invaluable in discerning biological molecular conformation.The emerging practice of localization in wireless sensor networks, the global positioning system (GPS), and distance-based pattern recognitionwill certainly simplify and benefit from this theory.We study the pervasive convex Euclidean bodies and their various representations.In particular, we make convex polyhedra, cones, and dual cones more visceral through illustration, andwe study the geometric relation of polyhedral cones to nonorthogonal bases biorthogonal expansion.We explain conversion between halfspace- and vertex-descriptions of convex cones,we provide formulae for determining dual cones,and we show how classic alternative systems of linear inequalities or linear matrix inequalities and optimality conditions can be explained by generalized inequalities in terms of convex cones and their duals.The conic analogue to linear independence, called conic independence, is introducedas a new tool in the study of classical cone theory; the logical next step in the progression:linear, affine, conic.Any convex optimization problem has geometric interpretation.This is a powerful attraction: the ability to visualize geometry of an optimization problem.We provide tools to make visualization easier.The concept of faces, extreme points, and extreme directions of convex Euclidean bodiesis explained here, crucial to understanding convex optimization.The convex cone of positive semidefinite matrices, in particular, is studied in depth.We mathematically interpret, for example,its inverse image under affine transformation, and we explainhow higher-rank subsets of its boundary united with its interior are convex.The Chapter on "Geometry of convex functions",observes analogies between convex sets and functions:The set of all vector-valued convex functions is a closed convex cone.Included among the examples in this chapter, we show how the real affinefunction relates to convex functions as the hyperplane relates to convex sets.Here, also, pertinent results formultidimensional convex functions are presented that are largely ignored in the literature;tricks and tips for determining their convexityand discerning their geometry, particularly with regard to matrix calculus which remains largely unsystematizedwhen compared with the traditional practice of ordinary calculus.Consequently, we collect some results of matrix differentiation in the appendices.The Euclidean distance matrix (EDM) is studied,its properties and relationship to both positive semidefinite and Gram matrices.We relate the EDM to the four classical axioms of the Euclidean metric;thereby, observing the existence of an infinity of axioms of the Euclidean metric beyondthe triangle inequality. We proceed byderiving the fifth Euclidean axiom and then explain why furthering this endeavoris inefficient because the ensuing criteria (while describing polyhedra)grow linearly in complexity and number.Some geometrical problems solvable via EDMs,EDM problems posed as convex optimization, and methods of solution arepresented;\eg, we generate a recognizable isotonic map of the United States usingonly comparative distance information (no distance information, only distance inequalities).We offer a new proof of the classic Schoenberg criterion, that determines whether a candidate matrix is an EDM. Our proofrelies on fundamental geometry; assuming, any EDM must correspond to a list of points contained in some polyhedron(possibly at its vertices) and vice versa.It is not widely known that the Schoenberg criterion implies nonnegativity of the EDM entries; proved here.We characterize the eigenvalues of an EDM matrix and then devisea polyhedral cone required for determining membership of a candidate matrix(in Cayley-Menger form) to the convex cone of Euclidean distance matrices (EDM cone); \ie,a candidate is an EDM if and only if its eigenspectrum belongs to a spectral cone for EDM^N.We will see spectral cones are not unique.In the chapter "EDM cone", we explain the geometric relationship betweenthe EDM cone, two positive semidefinite cones, and the elliptope.We illustrate geometric requirements, in particular, for projection of a candidate matrixon a positive semidefinite cone that establish its membership to the EDM cone. The faces of the EDM cone are described,but still open is the question whether all its faces are exposed as they are for the positive semidefinite cone.The classic Schoenberg criterion, relating EDM and positive semidefinite cones, isrevealed to be a discretized membership relation (a generalized inequality, a new Farkas''''''-like lemma)between the EDM cone and its ordinary dual. A matrix criterion for membership to the dual EDM cone is derived thatis simpler than the Schoenberg criterion.We derive a new concise expression for the EDM cone and its dual involvingtwo subspaces and a positive semidefinite cone."Semidefinite programming" is reviewedwith particular attention to optimality conditionsof prototypical primal and dual conic programs,their interplay, and the perturbation method of rank reduction of optimal solutions(extant but not well-known).We show how to solve a ubiquitous platonic combinatorial optimization problem from linear algebra(the optimal Boolean solution x to Ax=b)via semidefinite program relaxation.A three-dimensional polyhedral analogue for the positive semidefinite cone of 3X3 symmetricmatrices is introduced; a tool for visualizing in 6 dimensions.In "EDM proximity"we explore methods of solution to a few fundamental and prevalentEuclidean distance matrix proximity problems; the problem of finding that Euclidean distance matrix closestto a given matrix in the Euclidean sense.We pay particular attention to the problem when compounded with rank minimization.We offer a new geometrical proof of a famous result discovered by Eckart \& Young in 1936 regarding Euclideanprojection of a point on a subset of the positive semidefinite cone comprising all positive semidefinite matriceshaving rank not exceeding a prescribed limit rho.We explain how this problem is transformed to a convex optimization for any rank rho.

This book presents state-of-the-art results and methodologies in modern global optimization, and has been a staple reference for researchers, engineers, advanced students (also in applied mathematics), and practitioners in various fields of engineering. The second edition has been brought up to date and continues to develop a coherent and rigorous theory of deterministic global optimization, highlighting the essential role of convex analysis. The text has been revised and expanded to meet the needs of research, education, and applications for many years to come. Updates for this new edition include: · Discussion of modern approaches to minimax, fixed point, and equilibrium theorems, and to nonconvex optimization; · Increased focus on dealing more efficiently with ill-posed problems of global optimization, particularly those with hard constraints; · Important discussions of decomposition methods for specially structured problems; · A complete revision of the chapter on nonconvex quadratic programming, in order to encompass the advances made in quadratic optimization since publication of the first edition. · Additionally, this new edition contains entirely new chapters devoted to monotonic optimization, polynomial optimization and optimization under equilibrium constraints, including bilevel programming, multiobjective programming, and optimization with variational inequality constraint. From the reviews of the first edition: The book gives a good review of the topic. ...The text is carefully constructed and well written, the exposition is clear. It leaves a remarkable impression of the concepts, tools and techniques in global optimization. It might also be used as a basis and guideline for lectures on this subject. Students as well as professionals will profitably read and use it.—Mathematical Methods of Operations Research, 49:3 (1999)

Nonlinear analysis, formerly a subsidiary of linear analysis, has advanced as an individual discipline, with its own methods and applications. Moreover, students can now approach this highly active field without the preliminaries of linear analysis. As this text demonstrates, the concepts of nonlinear analysis are simple, their proofs direct, and their applications clear. No prerequisites are necessary beyond the elementary theory of Hilbert spaces; indeed, many of the most interesting results lie in Euclidean spaces. In order to remain at an introductory level, this volume refrains from delving into technical difficulties and sophisticated results not in current use. Applications are explained as soon as possible, and theoretical aspects are geared toward practical use. Topics range from very smooth functions to nonsmooth ones, from convex variational problems to nonconvex ones, and from economics to mechanics. Background notes, comments, bibliography, and indexes supplement the text.

An accessible introduction to convex algebraic geometry and semidefinite optimization. For graduate students and researchers in mathematics and computer science.

Vector Variational Inequalities and Vector Optimization

Convex Optimization Theory

Theory and Applications

This book provides the foundations of the theory of nonlinear optimization as well as some related algorithms and presents a variety of applications from diverse areas of applied sciences. The author combines three pillars of optimization?theoretical and algorithmic foundation, familiarity with various applications, and the ability to apply the theory and algorithms on actual problems?and rigorously and gradually builds the connection between theory, algorithms, applications, and implementation. Readers will find more than 170 theoretical, algorithmic, and numerical exercises that deepen and enhance the reader's understanding of the topics. The author includes offers several subjects not typically found in optimization books?for example, optimality conditions in sparsity-constrained optimization, hidden convexity, and total least squares. The book also offers a large number of applications discussed theoretically and algorithmically, such as circle fitting, Chebyshev center, the Fermat?Weber problem, denoising, clustering, total least squares, and orthogonal regression and theoretical and algorithmic topics demonstrated by the MATLAB? toolbox CVX and a package of m-files that is posted on the book?s web site. The results presented in this book originate from the last decade research work of the author in the ?eld of duality theory in convex optimization. The reputation of duality in the optimization theory comes mainly from the major role that it plays in formulating necessary and suf?cient optimality conditions and, consequently, in generatingdifferent algorithmic approachesfor solving mathematical programming problems. The investigations made in this work prove the importance of the duality theory beyond these aspects and emphasize its strong connections with different topics in convex analysis, nonlinear analysis, functional analysis and in the theory of monotone operators. The ?rst part of the book brings to the attention of the reader the perturbation approach as a fundamental tool for developing the so-called conjugate duality t- ory. The classical Lagrange and Fenchel duality approaches are particular instances of this general concept. More than that, the generalized interior point regularity conditions stated in the past for the two mentioned situations turn out to be p- ticularizations of the ones given in this general setting. In our investigations, the perturbationapproachrepresentsth estartingpointforderivingnewdualityconcepts for several classes of convex optimization problems. Moreover, via this approach, generalized Moreau–Rockafellar formulae are provided and, in connection with them, a new class of regularity conditions, called closedness-type conditions, for both stable strong duality and strong duality is introduced. By stable strong duality we understand the situation in which strong duality still holds whenever perturbing the objective function of the primal problem with a linear continuous functional.

Here is a book devoted to well-structured and thus efficiently solvable convex optimization problems, with emphasis on conic quadratic and semidefinite programming. The authors present the basic theory underlying these problems as well as their numerous applications in engineering, including synthesis of filters, Lyapunov stability analysis, and structural design. The authors also discuss the complexity issues and provide an overview of the basic theory of state-of-the-art polynomial time interior point methods for linear, conic quadratic, and semidefinite programming. The book's focus on well-structured convex problems in conic form allows for unified theoretical and algorithmical treatment of a wide spectrum of important optimization problems arising in applications.