Continuous Martingales And Brownian Motion

This textbook offers an approachable introduction to stochastic processes that explores the four pillars of random walk, branching processes, Brownian motion, and martingales. Building from simple examples, the authors focus on developing context and intuition before formalizing the theory of each topic. This inviting approach illuminates the key ideas and computations in the proofs, forming an ideal basis for further study. Consisting of many short chapters, the book begins with a comprehensive account of the simple random walk in one dimension. From here, different paths may be chosen according to interest. Themes span Poisson processes, branching processes, the Kolmogorov–Chentsov theorem, martingales, renewal theory, and Brownian motion. Special topics follow, showcasing a selection of important contemporary applications, including mathematical finance, optimal stopping, ruin theory, branching random walk, and equations of fluids. Engaging exercises accompany the theory throughout. Random Walk, Brownian Motion, and Martingales is an ideal introduction to the rigorous study of stochastic processes. Students and instructors alike will appreciate the accessible, example-driven approach. A single, graduate-level course in probability is assumed.

Stochastic processes occur everywhere in the sciences, economics and engineering, and they need to be understood by (applied) mathematicians, engineers and scientists alike. This book gives a gentle introduction to Brownian motion and stochastic processes, in general. Brownian motion plays a special role, since it shaped the whole subject, displays most random phenomena while being still easy to treat, and is used in many real-life models. Im this new edition, much material is added, and there are new chapters on "Wiener Chaos and Iterated Itô Integrals" and "Brownian Local Times".

A highly readable introduction to stochastic integration and stochastic differential equations, this book combines developments of the basic theory with applications. It is written in a style suitable for the text of a graduate course in stochastic calculus, following a course in probability. Using the modern approach, the stochastic integral is defined for predictable integrands and local martingales; then Itô's change of variable formula is developed for continuous martingales. Applications include a characterization of Brownian motion, Hermite polynomials of martingales, the Feynman-Kac functional and Schrödinger equation. For Brownian motion, the topics of local time, reflected Brownian motion, and time change are discussed. New to the second edition are a discussion of the Cameron-Martin-Girsanov transformation and a final chapter which provides an introduction to stochastic differential equations, as well as many exercises for classroom use. This book will be a valuable resource to all mathematicians, statisticians, economists, and engineers employing the modern tools of stochastic analysis. The text also proves that stochastic integration has made an important impact on mathematical progress over the last decades and that stochastic calculus has become one of the most powerful tools in modern probability theory. —Journal of the American Statistical Association [horizontal dagger separator] An attractive text...written in [a] lean and precise style...eminently readable. Especially pleasant are the care and attention devoted to details... A very fine book. —Mathematical Reviews

The most fundamental concepts in the theory of stochastic processes are the Markov property and the martingale property. This book is written for those who are familiar with both of these ideas in the discrete-time setting, and who now wish to explore stochastic processes in the continuous-time context. It has been our goal to write a systematic and thorough exposition of this subject, leading in many instances to the frontiers of knowledge. At the same time, we have endeavored to keep the mathematical prerequisities as low as possible, namely, knowledge of measure-theoretic probability and some acquaintance with discrete-time processes. The vehicle we have chosen for this task is Brownian motion, which we present as the canonical example of both a Markov process and a martingale in continuous time. We support this point of view by showing how by means of stochastic integration and random time change, all continuous martingales and a multitude of continuous Markov processes can be represented in terms of Brownian motion. This approach forces us to leave aside those processes which do not have continuous paths. Thus, the Poisson process is not a primary object of study, although it is developed to be used as a tool when we later study passage times of Brownian motion.

Brownian Motion

Probability with Martingales

Exponential Functionals of Brownian Motion and Related Processes

A Guide to Random Processes and Stochastic Calculus

Random Walk, Brownian Motion, and Martingales

Continuous Martingales and Brownian MotionSpringer Science & Business Media

Markov processes are among the most important stochastic processes for both theory and applications. This book develops the general theory of these processes, and applies this theory to various special examples. The initial chapter is devoted to the most important classical example - one dimensional Brownian motion. This, together with a chapter on continuous time Markov chains, provides the motivation for the general setup based on semigroups and generators. Chapters on stochastic calculus and probabilistic potential theory give an introduction to some of the key areas of application of Brownian motion and its relatives. A chapter on interacting particle systems treats a more recently developed class of Markov processes that have as their origin problems in physics and biology. This is a textbook for a graduate course that can follow one that covers basic probabilistic limit theorems and discrete time processes.

This classic introduction to probability theory for beginning graduate students covers laws of large numbers, central limit theorems, random walks, martingales, Markov chains, ergodic theorems, and Brownian motion. It is a comprehensive treatment concentrating on the results that are the most useful for applications. Its philosophy is that the best way to learn probability is to see it in action, so there are 200 examples and 450 problems. The fourth edition begins with a short chapter on measure theory to orient readers new to the subject.

Emphasizing fundamental mathematical ideas rather than proofs, Introduction to Stochastic Processes, Second Edition provides quick access to important foundations of probability theory applicable to problems in many fields. Assuming that you have a reasonable level of computer literacy, the ability to write simple programs, and the access to software for linear algebra computations, the author approaches the problems and theorems with a focus on stochastic processes evolving with time, rather than a particular emphasis on measure theory. For those lacking in exposure to linear differential and difference equations, the author begins with a brief introduction to these concepts. He proceeds to discuss Markov chains, optimal stopping, martingales, and Brownian motion. The book concludes with a chapter on stochastic integration. The author supplies many basic, general examples and provides exercises at the end of each chapter. New to the Second Edition: Expanded chapter on stochastic integration that introduces modern mathematical finance Introduction of Girsanov transformation and the Feynman-Kac formula Expanded discussion of It ô 's formula and the Black-Scholes formula for pricing options New topics such as Doob's maximal inequality and a discussion on self similarity in the chapter on Brownian motion Applicable to the fields of mathematics, statistics, and engineering as well as computer science, economics, business, biological science, psychology, and engineering, this concise introduction is an excellent resource both for students and professionals.

Lectures on Stochastic Analysis: Diffusion Theory

Brownian Motion Calculus

A Festschrift for Herman Rubin

Semimartingale Theory and Stochastic Calculus

Local Times and Excursion Theory for Brownian Motion

This book is a thorough and self-contained treatise of martingales as a tool in stochastic analysis, stochastic integrals and stochastic differential equations. The book is clearly written and details of proofs are worked out.

This book is based on a course given at Massachusetts Institute of Technology. It is intended to be a reasonably self-contained introduction to stochastic analytic techniques that can be used in the study of certain problems. The central theme is the theory of diffusions. In order to emphasize the intuitive aspects of probabilistic techniques, diffusion theory is presented as a natural generalization of the flow generated by a vector field. Essential to the development of this idea is the introduction of martingales and the formulation of diffusion theory in terms of martingales. The book will make valuable reading for advanced students in probability theory and analysis and will be welcomed as a concise account of the subject by research workers in these fields.

This book offers a rigorous and self-contained presentation of stochastic integration and stochastic calculus within the general framework of continuous semimartingales. The main tools of stochastic calculus, including Itô's formula, the optional stopping theorem and Girsanov's theorem, are treated in detail alongside many illustrative examples. The book also contains an introduction to Markov processes, with applications to solutions of stochastic differential equations and to connections between Brownian motion and partial differential equations. The theory of local times of semimartingales is discussed in the last chapter. Since its invention by Itô, stochastic calculus has proven to be one of the most important techniques of modern probability theory, and has been used in the most recent theoretical advances as well as in applications to other fields such as mathematical finance. Brownian Motion, Martingales, and Stochastic Calculus provides a strong theoretical background to the reader interested in such developments. Beginning graduate or advanced undergraduate students will benefit from this detailed approach to an essential area of probability theory. The emphasis is on concise and efficient presentation, without any concession to mathematical rigor. The material has been taught by the author for several years in graduate courses at two of the most prestigious French universities. The fact that proofs are given with full details makes the book particularly suitable for self-study. The numerous exercises help the reader to get acquainted with the tools of stochastic calculus.

This monograph is an introduction to some aspects of stochastic analysis in the framework of normal martingales, in both discrete and continuous time. The text is mostly self-contained, except for Section 5.7 that requires some background in geometry, and should be accessible to graduate students and researchers having already received a basic training in probability. Prereq- sites are mostly limited to a knowledge of measure theory and probability, namely?-algebras,expectations,andconditionalexpectations.Ashortint- duction to stochastic calculus for continuous and jump processes is given in Chapter 2 using normal martingales, whose predictable quadratic variation is the Lebesgue measure. There already exists several books devoted to stochastic analysis for c- tinuous di?usion processes on Gaussian and Wiener spaces, cf. e.g. [51], [63], [65], [72], [83], [84], [92], [128], [134], [143], [146], [147]. The particular f- ture of this text is to simultaneously consider continuous processes and jump processes in the uni?ed framework of normal martingales.

Methods of Mathematical Finance

Diffusions, Markov Processes, and Martingales: Volume 1, Foundations

Markov Processes from K. Itô's Perspective (AM-155)

Theory and Examples

A Tale of Wiener and Itô Measures

A graduate-course text, written for readers familiar with measure-theoretic probability and discrete-time processes, wishing to explore stochastic processes in continuous time. The vehicle chosen for this exposition is Brownian motion, which is presented as the canonical example of both a martingale and a Markov process with continuous paths. In this context, the theory of stochastic integration and stochastic calculus is developed, illustrated by results concerning representations of martingales and change of measure on Wiener space, which in turn permit a presentation of recent advances in financial economics. The book contains a detailed discussion of weak and strong solutions of stochastic differential equations and a study of local time for semimartingales, with special emphasis on the theory of Brownian local time. The whole is backed by a large number of problems and exercises.

In three chapters on Exponential Martingales, BMO-martingales, and Exponential of BMO, this book explains in detail the beautiful properties of continuous exponential martingales that play an essential role in various questions concerning the absolute continuity of probability laws of stochastic processes. The second and principal aim is to provide a full report on the exciting results on BMO in the theory of exponential martingales. The reader is assumed to be familiar with the general theory of continuous martingales.

This eagerly awaited textbook covers everything the graduate student in probability wants to know about Brownian motion, as well as the latest research in the area. Starting with the construction of Brownian motion, the book then proceeds to sample path properties like continuity and nowhere differentiability. Notions of fractal dimension are introduced early and are used throughout the book to describe fine properties of Brownian paths. The relation of Brownian motion and random walk is explored from several viewpoints, including a development of the theory of Brownian local times from random walk embeddings. Stochastic integration is introduced as a tool and an accessible treatment of the potential theory of Brownian motion clears the path for an extensive treatment of intersections of Brownian paths. An investigation of exceptional points on the Brownian path and an appendix on SLE processes, by Oded Schramm and Wendelin Werner, lead directly to recent research themes.

It has been 15 years since the first edition of Stochastic Integration and Differential Equations, A New Approach appeared, and in those years many other texts on the same subject have been published, often with connections to applications, especially mathematical finance. Yet in spite of the apparent simplicity of approach, none of these books has used the functional analytic method of presenting semimartingales and stochastic integration. Thus a 2nd edition seems worthwhile and timely, though it is no longer appropriate to call it "a new approach". The new edition has several significant changes, most prominently the addition of exercises for solution. These are intended to supplement the text, but lemmas needed in a proof are never relegated to the exercises. Many of the exercises have been tested by graduate students at Purdue and Cornell Universities. Chapter 3 has been completely redone, with a new, more intuitive and simultaneously elementary proof of the fundamental Doob-Meyer decomposition theorem, the more general version of the Girsanov theorem due to Lenglart, the Kazamaki-Novikov criteria for exponential local martingales to be martingales, and a modern treatment of compensators. Chapter 4 treats sigma martingales (important in finance theory) and gives a more comprehensive treatment of martingale representation, including both the Jacod-Yor theory and Emery's examples of martingales that actually have martingale representation (thus going beyond the standard cases of Brownian motion and the compensated Poisson process). New topics added include an introduction to the theory of the expansion of filtrations, a treatment of the Fefferman martingale inequality, and that the dual space of the martingale space H^1 can be identified with BMO martingales. Solutions to selected exercises are available at the web site of the author, with current URL http://www.orie.cornell.edu/~protter/books.html.

With Normal Martingales

Introduction to Stochastic Processes

An Introduction

Continuous Martingales and Brownian Motion

An Introduction Through Theory and Exercises

This book presents a concise treatment of stochastic calculus and its applications. It gives a simple but rigorous treatment of the subject including a range of advanced topics, it is useful for practitioners who use advanced theoretical results. It covers advanced applications, such as models in mathematical finance, biology and engineering.Self-contained and unified in presentation, the book contains many solved examples and exercises. It may be used as a textbook by advanced undergraduates and graduate students in stochastic calculus and financial mathematics. It is also suitable for practitioners who wish to gain an understanding or working knowledge of the subject. For mathematicians, this book could be a first text on stochastic calculus; it is good companion to more advanced texts by a way of examples and exercises. For people from other fields, it provides a way to gain a working knowledge of stochastic calculus. It shows all readers the applications of stochastic calculus methods and takes readers to the technical level required in research and sophisticated modelling.This second edition contains a new chapter on bonds, interest rates and their options. New materials include more worked out examples in all chapters, best estimators, more results on change of time, change of measure, random measures, new results on exotic options, FX options, stochastic and implied volatility, models of the age-dependent branching process and the stochastic Lotka-Volterra model in biology, non-linear filtering in engineering and five new figures.Instructors can obtain slides of the text from the author.

"This is a magnificent book! Its purpose is to describe in considerable detail a variety of techniques used by probabilists in the investigation of problems concerning Brownian motion....This is THE book for a capable graduate student starting out on research in probability: the effect of working through it is as if the authors are sitting beside one, enthusiastically explaining the theory, presenting further developments as exercises." –BULLETIN OF THE L.M.S.

Following the publication of the Japanese edition of this book, several inter esting developments took place in the area. The author wanted to describe some of these, as well as to offer suggestions concerning future problems which he hoped would stimulate readers working in this field. For these reasons, Chapter 8 was added. Apart from the additional chapter and a few minor changes made by the author, this translation closely follows the text of the original Japanese edition. We would like to thank Professor J. L. Doob for his helpful comments on the English edition. T. Hida T. P. Speed v Preface The physical phenomenon described by Robert Brown was the complex and erratic motion of grains of pollen suspended in a liquid. In the many years which have passed since this description, Brownian motion has become an object of study in pure as well as applied mathematics. Even now many of its important properties are being discovered, and doubtless new and useful aspects remain to be discovered. We are getting a more and more intimate understanding of Brownian motion.

Probability theory is nowadays applied in a huge variety of fields including physics, engineering, biology, economics and the social sciences. This book is a modern, lively and rigorous account which has Doob's theory of martingales in discrete time as its main theme. It proves important results such as Kolmogorov's Strong Law of Large Numbers and the Three-Series Theorem by martingale techniques, and the Central Limit Theorem via the use of characteristic functions. A distinguishing feature is its determination to keep the probability flowing at a nice tempo. It achieves this by being selective rather than encyclopaedic, presenting only what is essential to understand the fundamentals; and it assumes certain key results from measure theory in the main text. These measure-theoretic results are proved in full in appendices, so that the book is completely self-contained. The book is written for students, not for researchers, and has evolved through several years of class testing. Exercises play a vital rôle. Interesting and challenging problems, some with hints, consolidate what has already been learnt, and provide motivation to discover more of the subject than can be covered in a single introduction.

A Graduate Course in Stochastic Processes

Introduction to the Theory of Random Processes

Part II: Some Recent Martingale Problems

Stochastic Calculus

Brownian Motion and Stochastic Calculus

Brownian motion is one of the most important stochastic processes in continuous time and with continuous state space. Within the realm of stochastic processes, Brownian motion is at the intersection of Gaussian processes, martingales, Markov processes, diffusions and random fractals, and it has influenced the study of these topics. Its central position within mathematics is matched by numerous applications in science, engineering and mathematical finance. Often textbooks on probability theory cover, if at all, Brownian motion only briefly. On the other hand, there is a considerable gap to more specialized texts on Brownian motion which is not so easy to overcome for the novice. The authors' aim was to write a book which can be used as an introduction to Brownian motion and stochastic calculus, and as a first course in continuous-time and continuous-state Markov processes. They also wanted to have a text which would be both a readily accessible mathematical back-up for contemporary applications (such as mathematical finance) and a foundation to get easy access to advanced monographs. This textbook, tailored to the needs of graduate and advanced undergraduate students, covers Brownian motion, starting from its elementary properties, certain distributional aspects, path properties, and leading to stochastic calculus based on Brownian motion. It also includes numerical recipes for the simulation of Brownian motion.

Kiyosi Itô's greatest contribution to probability theory may be his introduction of stochastic differential equations to explain the Kolmogorov-Feller theory of Markov processes. Starting with the geometric ideas that guided him, this book gives an account of Itô's program. The modern theory of Markov processes was initiated by A. N. Kolmogorov. However, Kolmogorov's approach was too analytic to reveal the probabilistic foundations on which it rests. In particular, it hides the central role played by the simplest Markov processes: those with independent, identically distributed increments. To remedy this defect, Itô interpreted Kolmogorov's famous forward equation as an equation that describes the integral curve of a vector field on the space of probability measures. Thus, in order to show how Itô's thinking leads to his theory of stochastic integral equations, Stroock begins with an account of integral curves on the space of probability measures and then arrives at stochastic integral equations when he moves to a pathspace setting. In the first half of the book, everything is done in the context of general independent increment processes and without explicit use of Itô's stochastic integral calculus. In the second half, the author provides a systematic development of Itô's theory of stochastic integration: first for Brownian motion and then for continuous martingales. The final chapter presents Stratonovich's variation on Itô's theme and ends with an application to the characterization of the paths on which a diffusion is supported. The book should be accessible to readers who have mastered the essentials of modern probability theory and should provide such readers with a reasonably thorough introduction to continuous-time, stochastic processes.

Set-Indexed Martingales offers a unique, comprehensive development of a general theory of Martingales indexed by a family of sets. The authors establish-for the first time-an appropriate framework that provides a suitable structure for a theory of Martingales with enough generality to include many interesting examples. Developed from first principles, the theory brings together the theories of Martingales with a directed index set and set-indexed stochastic processes. Part One presents several classical concepts extended to this setting, including: stopping, predictability, Doob-Meyer decompositions, martingale characterizations of the set-indexed Poisson process, and Brownian motion. Part Two addresses convergence of sequences of set-indexed processes and introduces functional convergence for processes whose sample paths live in a Skorokhod-type space and semi-functional convergence for processes whose sample paths may be badly behaved. Completely self-contained, the theoretical aspects of this work are rich and promising. With its many important applications-especially in the theory of spatial statistics and in stochastic geometry- Set Indexed Martingales will undoubtedly generate great interest and inspire further research and development of the theory and applications.

This monograph discusses the existence and regularity properties of local times associated to a continuous semimartingale, as well as excursion theory for Brownian paths. Realizations of Brownian excursion processes may be translated in terms of the realizations of a Wiener process under certain conditions. With this aim in mind, the monograph presents applications to topics which are not usually treated with the same tools, e.g.: arc sine law, laws of functionals of Brownian motion, and the Feynman-Kac formula.

Stochastic Calculus and Financial Applications

A First Course in Stochastic Calculus

Probability

Introduction to Stochastic Integration

Some Aspects of Brownian Motion

Stochastic calculus has important applications to mathematical finance. This book will appeal to practitioners and students who want an elementary introduction to these areas. From the reviews: "As the preface says, ' This is a text with an attitude, and it is designed to reflect, wherever possible and appropriate, a prejudice for the concrete over the abstract ' . This is also reflected in the style of writing which is unusually lively for a mathematics book." --ZENTRALBLATT MATH

Now available in paperback, this celebrated book has been prepared with readers' needs in mind, remaining a systematic guide to a large part of the modern theory of Probability, whilst retaining its vitality. The authors' aim is to present the subject of Brownian motion not as a dry part of mathematical analysis, but to convey its real meaning and fascination. The opening, heuristic chapter does just this, and it is followed by a comprehensive and self-contained account of the foundations of theory of stochastic processes. Chapter 3 is a lively and readable account of the theory of Markov processes. Together with its companion volume, this book helps equip graduate students for research into a subject of great intrinsic interest and wide application in physics, biology, engineering, finance and computer science.

The following notes represent approximately the second half of the lectures I gave in the Nachdiplomvorlesung, in ETH, Zurich, between October 1991 and February 1992, together with the contents of six additional lectures I gave in ETH, in November and December 1993. Part I, the elder brother of the present book [Part II], aimed at the computation, as explicitly as possible, of a number of interesting functionals of Brownian motion. It may be natural that Part II, the younger brother, looks more into the main technique with which Part I was "working", namely: martingales and stochastic calculus. As F. Knight writes, in a review article on Part I, in which research on Brownian motion is compared to gold mining: "In the days of P. Levy, and even as late as the theorems of "Ray and Knight" (1963), it was possible for the practiced eye to pick up valuable reward without the aid of much technology . . . Thereafter, however, the rewards are increasingly achieved by the application of high technology". Although one might argue whether this golden age is really foregone, and discuss the "height" of the technology involved, this quotation is closely related to the main motivations of Part II: this technology, which includes stochastic calculus for general discontinuous semi-martingales, enlargement of filtrations, . . .

This volume collects papers about the laws of geometric Brownian motions and their time-integrals, written by the author and coauthors between 1988 and 1998. Throughout the volume, connections with more recent studies involving exponential functionals of Lévy processes are indicated. Some papers originally published in French are made available in English for the first time.

A Practical Introduction

Stochastic Analysis in Discrete and Continuous Settings

Stochastic Integration and Differential Equations

Continuous Exponential Martingales and BMO

An Introduction to Stochastic Processes

This book focuses on the probabilistic theory ofBrownian motion. This is a good topic to center a discussion around because Brownian motion is in the intersec tioll of many fundamental classes of processes. It is a continuous martingale, a Gaussian process, a Markov process or more specifically a process with in dependent increments; it can actually be defined, up to simple transformations, as the real-va process with independent increments and continuous paths. It is therefore no surprise that a vast array of techniques may be success fully applied to its study and we, consequently, chose to organize the book in the following way. After a first chapter where Brownian motion is introduced, each of the following ones is devoted to a new technique or notion and to some of its applications to Brownian motion. techniques, two are of para mount importance: stochastic calculus, the use ofwhich pervades the whole book and the powerful excursion theory, both of which are introduced in a self contained fashion and with a minimum of apparatus. They have made much easier the proofs of many results found in the epoch-making book of Itö and McKean: Diffusion Processes and their Sampie Paths, Springer (1965).

Brownian Motion Calculus presents the basics of Stochastic Calculus with a focus on the valuation of financial derivatives. It is intended as an accessible introduction to the technical literature. A clear distinction has been made between the mathematics that is convenient for a first introduction, and the more rigorous underpinnings which are best studied from the selected technical references. The inclusion out exercises makes the book attractive for self study. Standard probability theory and ordinary calculus are the prerequisites. Summary slides for revision and teaching can be found on the book website.

A highly readable introduction to stochastic integration and stochastic differential equations, this book combines developments of the basic theory with applications. It is written in a style suitable for the text of a graduate course in stochastic calculus, following a course in probability. Using the modern approach, the stochastic integral is defined for predictable integrands and local martingales; then It's chang formula is developed for continuous martingales. Applications include a characterization of Brownian motion, Hermite polynomials of martingales, the Feynman–Kac functional and the Schrödinger equation. For Brownian motion, the topics of local time, reflected Brownian motion, and time change are discussed. New to the second edition are a discussion of the Cameron–Martin–Girsanov transformation an chapter which provides an introduction to stochastic differential equations, as well as many exercises for classroom use. This book will be a valuable resource to all mathematicians, statisticians, economists, and engineers employing the modern tools of stochastic analysis. The text also proves that stochastic integration has made an important impact on mathematical progress over the last decades and tha has become one of the most powerful tools in modern probability theory. —Journal of the American Statistical Association An attractive text...written in [a] lean and precise style...eminently readable. Especially pleasant are the care and attention devoted to details... A very fine book. —Mathematical Reviews

This book concentrates on some general facts and ideas of the theory of stochastic processes. The topics include the Wiener process, stationary processes, infinitely divisible processes, and Ito stochastic equations. Basics of discrete time martingales are also presented and then used in one way or another throughout the book. Another common feature of the main body of the book is using stochastic integ random orthogonal measures. In particular, it is used for spectral representation of trajectories of stationary processes and for proving that Gaussian stationary processes with rational spectral densities are components of solutions to stochastic equations. In the case of infinitely divisible processes, stochastic integration allows for obtaining a representation of trajectories through jump measures. The Ito stoc also introduced as a particular case of stochastic integrals with respect to random orthogonal measures. Although it is not possible to cover even a noticeable portion of the topics listed above in a short book, it is hoped that after having followed the material presented here, the reader will have acquired a good understanding of what kind of results are available and what kind of techniques are used to obtai than 100 problems included, the book can serve as a text for an introductory course on stochastic processes or for independent study. Other works by this author published by the AMS include, Lectures on Elliptic and Parabolic Equations in Holder Spaces and Introduction to the Theory of Diffusion Processes.

Set-Indexed Martingales

Martingales And Stochastic Analysis

Brownian Motion, Martingales, and Stochastic Calculus

Continuous Time Markov Processes

A First Course in Stochastic Calculus is a complete guide for advanced undergraduate students to take the next step in exploring probability theory and for master's students in mathematical finance who would like to build an intuitive and theoretical understanding of stochastic processes. This book is also an essential tool for finance professionals who wish to sharpen their knowledge and intuition about stochastic calculus. Louis-Pierre Arguin offers an exceptionally clear introduction to Brownian motion and to random processes governed by the principles of stochastic calculus. The beauty and power of the subject are made accessible to readers with a basic knowledge of probability, linear algebra, and multivariable calculus. This is achieved by emphasizing numerical experiments using elementary Python coding to build intuition and adhering to a rigorous geometric point of view on the space of random variables. This unique approach is used to elucidate the properties of Gaussian processes, martingales, and diffusions. One of the book's highlights is a detailed and self-contained account of stochastic calculus applications to option pricing in finance. Louis-Pierre Arguin's masterly introduction to stochastic calculus seduces the reader with its quietly conversational style; even rigorous proofs seem natural and easy. Full of insights and intuition, reinforced with many examples, numerical projects, and exercises, this book by a prize-winning mathematician and great teacher fully lives up to the author's reputation. I give it my strongest possible recommendation. —Jim Gatheral, Baruch College I happen to be of a different persuasion, about how stochastic processes should be taught to undergraduate and MA students. But I have long been thinking to go against my own grain at some point and try to teach the subject at this level—together with its applications to finance—in one semester. Louis-Pierre Arguin's excellent and artfully designed text will give me the ideal vehicle to do so. —Ioannis Karatzas, Columbia University, New York

This sequel to Brownian Motion and Stochastic Calculus by the same authors develops contingent claim pricing and optimal consumption/investment in both complete and incomplete markets, within the context of Brownian-motion-driven asset prices. The latter topic is extended to a study of equilibrium, providing conditions for existence and uniqueness of market prices which support trading by several heterogeneous agents. Although much of the incomplete-market material is available in research papers, these topics are treated for the first time in a unified manner. The book contains an extensive set of references and notes describing the field, including topics not treated in the book. This book will be of interest to researchers wishing to see advanced mathematics applied to finance. The material on optimal consumption and investment, leading to equilibrium, is addressed to the theoretical finance community. The chapters on contingent claim valuation present techniques of practical importance, especially for pricing exotic options.

The first edition of this single volume on the theory of probability has become a highly-praised standard reference for many areas of probability theory. Chapters from the first edition have been revised and corrected, and this edition contains four new chapters. New material covered includes multivariate and ratio ergodic theorems, shift coupling, Palm distributions, Harris recurrence, invariant measures, and strong and weak ergodicity.

Semimartingale Theory and Stochastic Calculus presents a systematic and detailed account of the general theory of stochastic processes, the semimartingale theory, and related stochastic calculus. The book emphasizes stochastic integration for semimartingales, characteristics of semimartingales, predictable representation properties and weak convergence of semimartingales. It also includes a concise treatment of absolute continuity and singularity, contiguity, and entire separation of measures by semimartingale approach. Two basic types of processes frequently encountered in applied probability and statistics are highlighted: processes with independent increments and marked point processes encountered frequently in applied probability and statistics. Semimartingale Theory and Stochastic Calculus is a self-contained and comprehensive book that will be valuable for research mathematicians, statisticians, engineers, and students.

Introduction to Stochastic Calculus with Applications

Foundations of Modern Probability

This compact yet thorough text zeros in on the parts of the theory that are particularly relevant to applications . It begins with a description of Brownian motion and the associated stochastic calculus, including their relationship to partial differential equations. It solves stochastic differential equations by a variety of methods and studies in detail the one-dimensional case. The book concludes with a treatment of semigroups and generators, applying the theory of Harris chains to diffusions, and presenting a quick course in weak convergence of Markov chains to diffusions. The presentation is unparalleled in its clarity and simplicity. Whether your students are interested in probability, analysis, differential geometry or applications in operations research, physics, finance, or the many other areas to which the subject applies, you'll find that this text brings together the material you need to effectively and efficiently impart the practical background they need.

This book provides a comprehensive introduction to the theory of stochastic calculus and some of its applications. It is the only textbook on the subject to include more than two hundred exercises with complete solutions. After explaining the basic elements of probability, the author introduces more advanced topics such as Brownian motion, martingales and Markov processes. The core of the book covers stochastic calculus, including stochastic differential equations, the relationship to partial differential equations, numerical methods and simulation, as well as applications of stochastic processes to finance. The final chapter provides detailed solutions to all exercises, in some cases presenting various solution techniques together with a discussion of advantages and drawbacks of the methods used. Stochastic Calculus will be particularly useful to advanced undergraduate and graduate students wishing to acquire a solid understanding of the subject through the theory and exercises. Including full mathematical statements and rigorous proofs, this book is completely self-contained and suitable for lecture courses as well as self-study.