

## Basic Classes Of Linear Operators 1st Edition

Basic Classes of Linear OperatorsBirkh à user

Offers an examination of the multivariate approximation case Special focus on the Bernstein operators, including applications, and on two new classes of Bernstein-type operators Many general estimates, leaving room for future applications (e.g. the B-spline case) Extensions to approximation operators acting on spaces of vector functions Historical perspective in the form of previous significant results

The Operator Theory conferences, organized by the Department of Mathematics of INCREST and the University of Timi–ara, are conceived as a means to promote cooperation and exchange of information between specialists in all areas of operator theory. This volume consists of a careful selection of papers contributed by the participants of the 1986 Conference. They reflect most of the topics dealt with by the modern operator theory, including recent advances in dual operator algebras and the invariant subspace problem, operators in indefinite metric spaces, hyponormal, quasi triangular and decomposable operators, various problems in  $C^*$ - and  $W^*$ -algebras and so on. The research contracts of the Department of Mathematics of INCREST with the National Council for Science and Technology of Romania provided the means for developing the research activity in mathematics; they represent the generous framework of these meetings, too. It is our pleasure to acknowledge the financial support of UNESCO which also contributed to the success of this meeting. We are indebted to Professor Israel Gohberg for including these Proceedings in the OT Series and for valuable advice in the editing process. Birkhüser Verlag was very cooperative in publishing this volume. Camelia Minculescu, Iren Nemethi and Rodica Stoescu dealt with the difficult task of typing the whole manuscript using a Rank Xerox 860 word processor; we thank them for the excellent job they did.

Selected Topics in Complex Analysis

Approximation Theory Using Positive Linear Operators

Mathematical Methods in Physics

Special Classes of Linear Operators and Other Topics 11th International Conference on Operator Theory, Bucharest (romania), June 2-12, 1986

A comprehensive graduate textbook that introduces functional analysis with an emphasis on the theory of linear operators and its application to differential equations, integral equations, infinite systems of linear equations, approximation theory, and numerical analysis. As a textbook designed for senior undergraduate and graduate students, it begins with the geometry of Hilbert spaces and proceeds to the theory of linear operators on these spaces including Banach spaces. Presented as a natural continuation of linear algebra, the book provides a firm foundation in operator theory which is an essential part of mathematical training for students of mathematics, engineering, and other technical sciences.

Constructs a theoretical framework for the study of linear relations and provides underlying concepts, rules, formulae, theorems and techniques. The book compares the inversion, adjoints, completion and closure of various classes of linear operators. It highlights compact and precompact relations.

Most books on linear operators are not easy to follow for students and researchers without an extensive background in mathematics. Self-contained and using only matrix theory, Invitation to Linear Operators: From Matrices to Bounded Linear Operators on a Hilbert Space explains in easy-to-follow steps a variety of interesting recent results on linear operators on a Hilbert space. The author first states the important properties of a Hilbert space, then sets out the fundamental properties of bounded linear operators on a Hilbert space. The final section presents some of the more recent developments in bounded linear operators.

Introduction to Linear Operator Theory

Basic Operator Theory

Operator Theory, Analysis and the State Space Approach

Distributions, Hilbert Space Operators, Variational Methods, and Applications in Quantum Physics

The second edition of this textbook presents the basic mathematical knowledge and skills that are needed for courses on modern theoretical physics, such as those on quantum mechanics, classical and quantum field theory, and related areas. The authors stress that learning mathematical physics is not a passive process and include numerous detailed proofs, examples, and over 200 exercises, as well as hints linking mathematical concepts and theories. All of the material from the first edition has been updated, and five new chapters have been added on such topics as distributions, Hilbert space operators, and variational methods. The text is divided into three parts. - Part I: A brief introduction to (Schwartz) distribution theory. Elements from the theories of ultra distributions and (Fourier) hyperfunctions are given in addition to some deeper results for Schwartz distributions, thus providing a rather comprehensive introduction to the theory of generalized functions. Basic properties and methods for distributions are developed with applications to constant coefficient ODEs and PDEs. The relation between distributions and holomorphic functions is considered, as well as basic properties of Sobolev spaces. - Part II: Fundamental facts about Hilbert spaces. The basic theory of linear (bounded and unbounded) operators in Hilbert spaces and special classes of linear operators - compact, Hilbert-Schmidt, trace class, and Schrödinger operators, as needed in quantum physics and quantum information theory – are explored. This section also contains a detailed spectral analysis of all major classes of linear operators, including completeness of generalized eigenfunctions, as well as of (completely) positive mappings, in particular quantum operations. - Part III: Direct methods of the calculus of variations and their applications to boundary- and eigenvalue-problems for linear and nonlinear partial differential operators. The authors conclude with a discussion of the Hohenberg-Kohn variational principle. The appendices contain proofs of more general and deeper results, including completions, basic facts about metrizable Hausdorff locally convex topological vector spaces, Baire’s fundamental results and their main consequences, and bilinear functionals. Mathematical Methods in Physics is aimed at a broad community of graduate students in mathematics, mathematical physics, quantum information theory, physics and engineering, as well as researchers in these disciplines. Expanded content and relevant updates will make this new edition a valuable resource for those working in these disciplines.

This self-contained book provides the reader with a comprehensive presentation of recent investigations on operator theory over non-Archimedean Banach and Hilbert spaces. This includes, non-Archimedean valued fields, bounded and unbounded linear operators, bilinear forms, functions of linear operators and one-parameter families of bounded linear operators on free branch spaces.

The theories of quadratic forms and their applications appear in many parts of mathematics and the sciences. All students of mathematics have the opportunity to encounter such concepts and applications in their first course in linear algebra. This subject and its extensions to infinite dimensions comprise the theory of the numerical range  $W(T)$ . There are two competing names for  $W(T)$ , namely, the numerical range of  $T$  and the field of values for  $T$ . The former has been favored historically by the functional analysis community, the latter by the matrix analysis community. It is a toss-up to decide which is preferable, and we have finally chosen the former because it is our habit, it is a more efficient expression, and because in recent conferences dedicated to  $W(T)$ , even the linear algebra community has adopted it. Also, one universally refers to the numerical radius, and not to the field of values radius. Originally, Toeplitz and Hausdorff called it the Wertvorrat of a bilinear form, so other good names would be value field or form values. The Russian community has referred to it as the Hausdorff domain. Murnaghan in his early paper first called it the region of the complex plane covered by those values for an  $n \times n$  matrix  $T$ , then the range of values of a Hermitian matrix, then the field of values when he analyzed what he called the sought-for region.

Classes of linear operators on pseudo-Hilbert spaces and applications. 1

In Honor of Rien Kaashoek

11th International Conference on Operator Theory Bucharest (Romania) June 2–12, 1986

This book is a modern treatment of a classical area of operator theory. Written in a meticulous and detailed style, with the modern graduate student of analysis in mind, it contains many simplifications of existing literature. It is full of new results, as well as many illuminating examples. Carefully cross referenced throughout, it also includes an extensive list of the relevant literature.

This volume opens with a paper by V.P. Havin that presents a comprehensive survey of the work of mathematician S.Ya. Khavinson. It includes a complete bibliography, previously unpublished, of 163 items, and twelve peer-reviewed research and expository papers by leading mathematicians, including the joint paper by Khavinson and T.S. Kuzina. The emphasis is on the usage of tools from functional analysis, potential theory, algebra, and topology.

General spectral theory; Riesz operators; Hermitian operators; Prespectral operators; Well-bounded operators.

Classes of Linear Operators on Pseudo-Hilbert and Applications

Classes of Linear Operators

Linear Operators and Their Spectra

The S. Ya. Khavinson Memorial Volume

*This volume is dedicated to Rien Kaashoek on the occasion of his 80th birthday and celebrates his many contributions to the field of operator theory during more than fifty years. In the first part of the volume, biographical information and personal accounts on the life of Rien Kaashoek are presented. Eighteen research papers by friends and colleagues of Rien Kaashoek are included in the second part. Contributions by J. Agler, Z.A. Lykova, N.J. Young, J.A. Ball, G.J. Groenewald, S. ter Horst, H. Bart, T. Ehrhardt, B. Silbermann, J.M. Bogoya, S.M. Grudsky, I.S. Malysheva, A. Böttcher, E. Wegert, Z. Zhou, Y. Eidelman, I. Halmovici, A.E. Frazho, A.C.M. Ran, B. Fritzsche, B. Kirstein, C.Madler, J. J. Jaftha, D.B. Janse van Rensburg, P. Junghanns, R. Kaiser, J. Nemcova, M. Petreczky, J.H. van Schuppen, L. Plevnik, P. Semrl, A. Sakhnovich, F.-O. Speck, S. Srenac, H.J. Woerdeman, H. Molkowicz and N. Vasilevski.*

*This book is an introduction to the subject and is devoted to standard material on linear functional analysis, and presents some ergodic theorems for classes of operators containing the quasi-compact operators. It discusses various classes of operators connected with the numerical range.*

*This book provides the reader with a self-contained treatment of the classical operator theory with significant applications to abstract differential equations, and an elegant introduction to basic concepts and methods of the rapidly growing theory of the so-called p-adic operator theory.*

*Distributions, Hilbert Space Operators, and Variational Methods*

*From Matrices to Bounded Linear Operators on a Hilbert Space*

*The Field of Values of Linear Operators and Matrices*

*Linear Operators in Hilbert Spaces*

Physics has long been regarded as a wellspring of mathematical problems. Mathematical Methods in Physics is a self-contained presentation, driven by historic motivations, excellent examples, detailed proofs, and a focus on those parts of mathematics that are needed in more ambitious courses on quantum mechanics and classical and quantum field theory. Aimed primarily at a broad community of graduate students in mathematics, mathematical physics, physics and engineering, as well as researchers in these disciplines.

Authoritative text presenting a broad view of the spectral theory of non-self-adjoint linear operators.

After the book "Basic Operator Theory" by Gohberg-Goldberg was published, we, that is the present authors, intended to continue with another book which would show the readers the large variety of classes of operators and the important role they play in applications. The book was planned to be of modest size, but due to the profusion of results in this area of analysis, the number of topics grew larger than expected. Consequently, we decided to divide the material into two volumes - the first volume being presented now. During the past years, courses and seminars were given at our respective institutions based on parts of the texts. These were well received by the audience and enabled us to make appropriate choices for the topics and presentation for the two volumes. We would like to thank G.J. Groenewald, A.B. Kujper and A.C.M. Ran of the Vrije Universiteit at Amsterdam, who provided us with lists of remarks and corrections. We are now aware that the Basic Operator Theory book should be revised so that it may suitably fit in with our present volumes. This revision is planned to be the last step of an induction and not the first.

Numerical Range

Classes of Linear Operators Vol. 1 and 2

An Introduction to Classical and P-adic Theory of Linear Operators and Applications

Spectral Theory and Applications of Linear Operators and Block Operator Matrices

*Examining recent mathematical developments in the study of Fredholm operators, spectral theory and block operator matrices, with a rigorous treatment of classical Riesz theory of polynomially-compact operators, this volume covers both abstract and applied developments in the study of spectral theory. These topics are intimately related to the stability of underlying physical systems and play a crucial role in many branches of mathematics as well as numerous interdisciplinary applications. By studying classical Riesz theory of polynomially compact operators in order to establish the existence results of the second kind operator equations, this volume will assist the reader working to describe the spectrum, multiplicities and localization of the eigenvalues of polynomially-compact operators.*

*This book consists of three major parts. The first two parts deal with general mathematical concepts and certain areas of operator theory. The third part is devoted to ill-posed problems. It can be read independently of the first two parts and presents a good example of applying the methods of calculus and functional analysis. The first part "Basic Concepts" briefly introduces the language of set theory and concepts of abstract, linear and multilinear algebra. Also introduced are the language of topology and fundamental concepts of calculus: the limit, the differential, and the integral. A special section is devoted to analysis on manifolds. The second part "Operators" describes the most important function spaces and operator classes for both linear and nonlinear operators. Different kinds of generalized functions and their transformations are considered. Elements of the theory of linear operators are presented. Spectral theory is given a special focus. The third part "Ill-Posed Problems" is devoted to problems of mathematical physics, integral and operator equations, evolution equations and problems of integral geometry. It also deals with problems of analytic continuation. Detailed coverage of the subjects and numerous examples and exercises make it possible to use the book as a textbook on some areas of calculus and functional analysis. It can also be used as a reference textbook because of the extensive scope and detailed references with comments.*

*These two volumes constitute texts for graduate courses in linear operator theory. The reader is assumed to have a knowledge of both complex analysis and the first elements of operator theory. The texts are intended to concisely present a variety of classes of linear operators, each with its own character, theory, techniques and tools. For each of the classes, various differential and integral operators motivate or illustrate the main results. Although each class is treated separately and the first impression may be that of many different theories, interconnections appear frequently and unexpectedly. The result is a beautiful, unified and powerful theory. The classes we have chosen are representatives of the principal important classes of operators, and we believe that these illustrate the richness of operator theory, both in its theoretical developments and in its applicants. Because we wanted the books to be of reasonable size, we were selective in the classes we chose and restricted our attention to the main features of the corresponding theories. However, these theories have been updated and enhanced by new developments, many of which appear here for the first time in an operator-theory text. In the selection of the material the taste and interest of the authors played an important role.*

*Classes of linear operators on pseudo-Hilbert spaces and applications. 2*

*Invitation to Linear Operators*

*Special Classes of Linear Operators and Other Topics*

*Operator Theory and Ill-Posed Problems*

*Basic Operator Theory provides an introduction to functional analysis with an emphasis on the theory of linear operators and its application to differential and integral equations, approximation theory, and numerical analysis. A textbook designed for senior undergraduate and graduate students, Basic Operator Theory begins with the geometry of Hilbert space and proceeds to the spectral theory for compact self-adjoint operators with a wide range of applications. Part of the volume is devoted to Banach spaces and operators acting on these spaces. Presented as a natural continuation of linear algebra, Basic Operator Theory provides a firm foundation in operator theory, an essential part of mathematical training for students of mathematics, engineering, and other technical sciences.*

*This English edition is almost identical to the German original Lineare Operatoren in Hilberträumen, published by B. G. Teubner, Stuttgart in 1976. A few proofs have been simplified, some additional exercises have been included, and a small number of new results has been added (e.g., Theorem 11.11 and Theorem 11.23). In addition a great number of minor errors has been corrected. Frankfurt, January 1980 J. Weidmann vii Preface to the German edition The purpose of this book is to give an introduction to the theory of linear operators on Hilbert spaces and then to proceed to the interesting applications of differential operators to mathematical physics. Besides the usual introductory courses common to both mathematicians and physicists, only a fundamental knowledge of complex analysis and of ordinary differential equations is assumed. The most important results of Lebesgue integration theory, to the extent that they are used in this book, are compiled with complete proofs in Appendix A. I hope therefore that students from the fourth semester on will be able to read this book without major difficulty. However, it might also be of some interest and use to the teaching and research mathematician or physicist, since among other things it makes easily accessible several new results of the spectral theory of differential operators.*

*This book focuses on the theory of linear operators on non-Archimedean Banach spaces. The topics treated in this book range from a basic introduction to non-Archimedean valued fields, free non-Archimedean Banach spaces, bounded and unbounded linear operators in the non-Archimedean setting, to the spectral theory for some classes of linear operators. The theory of Fredholm operators is emphasized and used as an important tool in the study of the spectral theory of non-Archimedean operators. Explicit descriptions of the spectra of some operators are worked out. Moreover, detailed background materials on non-Archimedean valued fields and free non-Archimedean Banach spaces are included for completeness and for reference. The readership of the book is aimed toward graduate and postgraduate students, mathematicians, and non-mathematicians such as physicists and engineers who are interested in non-Archimedean functional analysis. Further, it can be used as an introduction to the study of non-Archimedean operator theory in general and to the study of spectral theory in other special cases.*

Vol. 2

Non-Archimedean Linear Operators and Applications

Perturbation theory for linear operators

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