

*An Introduction To Symplectic Geometry*

**A beautiful and comprehensive introduction to this important field. --Dusa McDuff, Barnard College, Columbia University This excellent book gives a detailed, clear, and wonderfully written treatment of the interplay between the world of Stein manifolds and the more topological and flexible world of Weinstein manifolds. Devoted to this subject with a long history, the book serves as a super introduction to this area and also contains the authors' new results. --Tomasz Mrowka, MIT This book is devoted to the interplay between complex and symplectic geometry in affine complex manifolds. Affine complex (a.k.a. Stein) manifolds have canonically built into them symplectic geometry which is responsible for many phenomena in complex geometry and analysis. The goal of the book is the exploration of this symplectic geometry (the road from ``Stein to Weinstein'') and its applications in the complex geometric world of Stein manifolds (the road ``back''). This is the first book which systematically explores this connection, thus providing a new approach to the classical subject of Stein manifolds. It also contains the first detailed investigation of Weinstein manifolds, the symplectic counterparts of Stein manifolds, which play an important role in symplectic and contact topology. Assuming only a general background from differential topology, the book provides introductions to the various techniques from the theory of functions of several complex variables, symplectic geometry,  $h^1$ -principles, and Morse theory that enter the proofs of the main results. The main results of the book are original results of the authors, and several of these results appear here for the first time. The book will be beneficial for all students and mathematicians interested in geometric aspects of complex analysis, symplectic and contact topology, and the interconnections between these subjects.**

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**This book offers a complete discussion of techniques and topics intervening in the mathematical treatment of quantum and semi-classical mechanics. It starts with a very readable introduction to symplectic geometry. Many topics are also of genuine interest for pure mathematicians working in geometry and topology.**

**The goal of these notes is to provide a fast introduction to symplectic geometry for graduate students with some knowledge of differential geometry, de Rham theory and classical Lie groups. This text addresses symplectomorphisms, local forms, contact manifolds, compatible almost complex structures, Kaehler manifolds, hamiltonian mechanics, moment maps, symplectic reduction and symplectic toric manifolds. It contains guided problems, called homework, designed to complement the exposition or extend the reader's understanding. There are by now excellent references on symplectic geometry, a subset of which is in the bibliography of this book. However, the most efficient introduction to a subject is often a short elementary treatment, and these notes attempt to serve that purpose. This text provides a taste of areas of current research and will prepare the reader to explore recent papers and extensive books on symplectic geometry where the pace is much faster. For this reprint numerous corrections and clarifications have been made, and the layout has been improved.**

**Differential geometry arguably offers the smoothest transition from the standard university mathematics sequence of the first four semesters in calculus, linear algebra, and differential equations to the higher levels of abstraction and proof encountered at the upper division by mathematics majors. Today it is possible to describe differential geometry as "the study of structures on the tangent space," and this text develops this point of view. This book, unlike other introductory texts in differential geometry, develops the architecture necessary to**

**introduce symplectic and contact geometry alongside its Riemannian cousin. The main goal of this book is to bring the undergraduate student who already has a solid foundation in the standard mathematics curriculum into contact with the beauty of higher mathematics. In particular, the presentation here emphasizes the consequences of a definition and the careful use of examples and constructions in order to explore those consequences.**

**Contact and Symplectic Topology**

**An Introduction to Symplectic Geometry**

**Symplectic Geometry**

**A Symplectic View of Physics**

**Structure of Dynamical Systems**

The book introduces the basic notions in Symplectic and Contact Geometry at the level of the second year graduate student. It also contains many exercises, some of which are solved only in the last chapter. We begin with the linear theory, then give the definition of symplectic manifolds and some basic examples, review advanced calculus, discuss Hamiltonian systems, tour rapidly group and the basics of contact geometry, and solve problems in chapter 8. The material just described can be used as a one semester course on Symplectic and Contact Geometry. The book contains also more advanced material, suitable to advanced graduate students and researchers. Contents: Symplectic Vector Spaces Symplectic Manifolds Hamiltonian Systems and Poisson Algebra Group Actions Contact Manifolds Solutions of Selected Exercises Epilogue: The  $C0$ -Symplectic and Contact Topology Readership: Graduate students, researchers and more advanced mathematicians. Symplectic; Contact Geometry Key Features: It is brief The easy part has been tested and been used for a short course The advanced material develops things related to one of the author's research further There is no book, going from the very elementary part to the very advanced level, like this one

This excellent book will be very useful for students and researchers wishing to learn the basics of Poisson geometry, as well as for those who know something about the subject but wish to update and deepen their knowledge. The authors' philosophy that Poisson geometry is an amalgam of foliation theory, symplectic geometry, and Lie theory enables them to organize the book in a very coherent way. –Alan Weinstein, University of California at Berkeley This well-written book is an excellent starting point for students and researchers who want to learn about the basics of Poisson geometry. The topics covered are fundamental to the theory and avoid any drift into specialized questions; they are illustrated through a large collection of instructive and interesting exercises. The book is ideal as a graduate textbook on the subject, but also for self-study. –Eckhard Meinrenken, University of Toronto Symplectic and contact geometry naturally emerged from the mathematical description of classical physics. The discovery of new rigidity phenomena and properties satisfied by these geometric structures launched a new research field worldwide. The intense activity of many European research groups in this field is reflected by the ESF Research Networking Programme "Contact And Symplectic Topology" (CAST). The lectures of the Summer School in Nantes (June 2011) and of the CAST Summer School in Budapest (July 2012) provide a nice panorama of many aspects of the present status of contact and symplectic topology. The notes of the minicourses offer a gentle introduction to topics which have developed in an amazing speed in the recent past. These topics include 3-dimensional and higher dimensional contact topology, Fukaya categories, asymptotically holomorphic methods in contact topology, bordered Floer homology, embedded contact homology, and flexibility results for Stein manifolds.

Symplectic geometry, well known as the basic structure of Hamiltonian mechanics, is also the foundation of optics. In fact, optical systems (geometric or wave) have an even richer symmetry structure than mechanical ones (classical or quantum). The symmetries underlying the geometric model of light are based on the symplectic group. Geometric Optics on Phase Space develops both geometric optics and group theory from first principles in their Hamiltonian formulation on phase space. This treatise provides the mathematical background and also collects a host of useful methods of practical importance, particularly the fractional Fourier transform currently used for image processing. The reader will appreciate the beautiful similarities between Hamilton's mechanics and this approach to optics. The appendices link the geometry thus introduced to wave optics through Lie methods. The book addresses researchers and graduate students.

**Introduction to Mechanics and Symmetry**

**An Introduction to Lie Groups and Lie Algebras**

**Lectures on Symplectic Geometry**

**From Stein to Weinstein and Back**

**Riemannian, Contact, Symplectic**

"Symplectic Geometric Algorithms for Hamiltonian Systems" will be useful not only for numerical analysts, but also for those in theoretical physics, computational chemistry, celestial mechanics, etc. The book generalizes and develops the generating function and Hamilton-Jacobi equation theory from the perspective of the symplectic geometry and symplectic algebra. It will be a useful resource for engineers and scientists in the fields of quantum theory, astrophysics, atomic and molecular dynamics, climate prediction, oil exploration, etc. Therefore a systematic research and development of numerical methodology for Hamiltonian systems is well motivated. Were it successful, it would imply wide-ranging applications.

This book is devoted to pseudo-holomorphic curve methods in symplectic geometry. It contains an introduction to symplectic geometry and relevant techniques of Riemannian geometry, proofs of Gromov's compactness theorem, an investigation of local properties of holomorphic curves, including positivity of intersections, and applications to Lagrangian embeddings problems. The chapters are based on a series of lectures given previously by the authors M. Audin, A. Banyaga, P. Gauduchon, F. Labourie, J. Lafontaine, F. Lalonde, Gang Liu,

D. McDuff, M.-P. Muller, P. Pansu, L. Polterovich, J.C. Sikorav. In an attempt to make this book accessible also to graduate students, the authors provide the necessary examples and techniques needed to understand the applications of the theory. The exposition is essentially self-contained and includes numerous exercises. Symplectic geometry is a central topic of current research in mathematics. Indeed, symplectic methods are key ingredients in the study of dynamical systems, differential equations, algebraic geometry, topology, mathematical physics and representations of Lie groups. This book is a true introduction to symplectic geometry, assuming only a general background in analysis and familiarity with linear algebra. It starts with the basics of the geometry of symplectic vector spaces. Then, symplectic manifolds are defined and explored. In addition to the essential classic results, such as Darboux's theorem, more recent results and ideas are also included here, such as symplectic capacity and pseudoholomorphic curves. These ideas have revolutionized the subject. The main examples of symplectic manifolds are given, including the cotangent bundle, Kahler manifolds, and coadjoint orbits. Further principal ideas are carefully examined, such as Hamiltonian vector fields, the Poisson bracket, and connections with contact manifolds. Berndt describes some of the close connections between symplectic geometry and mathematical physics in the last two chapters of the book. In particular, the moment map is defined and explored, both mathematically and in its relation to physics. He also introduces symplectic reduction, which is an important tool for reducing the number of variables in a physical system and for constructing new symplectic manifolds from old. The final chapter is on quantization, which uses symplectic methods to take classical mechanics to quantum mechanics. This section includes a discussion of the Heisenberg group and the Weil (or metaplectic) representation of the symplectic group. Several appendices provide background material on vector bundles, on cohomology, and on Lie groups and Lie algebras and their representations. Berndt's presentation of symplectic geometry is a clear and concise introduction to the major methods and applications of the subject, and requires only a minimum of prerequisites. This book would be an excellent text for a graduate course or as a source for anyone who wishes to learn about symplectic geometry.

Symplectic geometry has its origins as a geometric language for classical mechanics. But it has recently exploded into an independent field interconnected with many other areas of mathematics and physics. The goal of the IAS/Park City Mathematics Institute Graduate Summer School on Symplectic Geometry and Topology was to give an intensive introduction to these exciting areas of current research. Included in this proceedings are lecture notes from the following courses: Introduction to Symplectic Topology by D. McDuff; Holomorphic Curves and Dynamics in Dimension Three by H. Hofer; An Introduction to the Seiberg-Witten Equations on Symplectic Manifolds by C. Taubes; Lectures on Floer Homology by D. Salamon; A Tutorial on Quantum Cohomology by A. Givental; Euler Characteristics and Lagrangian Intersections by R. MacPherson; Hamiltonian Group Actions and Symplectic Reduction by L. Jeffrey; and Mechanics: Symmetry and Dynamics by J. Marsden. Information for our distributors: Titles in this series are copublished with the Institute for Advanced Study/Park City Mathematics Institute. Members of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) receive a 20% discount from list price.

A Brief Introduction to Symplectic and Contact Manifolds

An Introduction based on the Seminar in Bern, 1992

Symplectic Geometry and Topology

Locally Conformal Kähler Geometry

A Basic Exposition of Classical Mechanical Systems

Among all the Hamiltonian systems, the integrable ones have special geometric properties; in particular, their solutions are very regular and quasi-periodic. This book serves as an introduction to symplectic and contact geometry for graduate students, exploring the underlying geometry of integrable Hamiltonian systems. Includes exercises designed to complement the exposition, and up-to-date references.

This introductory book offers a unique and unified overview of symplectic geometry, highlighting the differential properties of symplectic manifolds. It consists of six chapters: Some Algebra Basics, Symplectic Manifolds, Cotangent Bundles, Symplectic  $G$ -spaces, Poisson Manifolds, and A Graded Case, concluding with a discussion of the differential properties of graded symplectic manifolds of dimensions  $(0, n)$ . It is a useful reference resource for students and researchers interested in geometry, group theory, analysis and differential equations. This book is also inspiring in the emerging field of Geometric Science of Information, in particular the chapter on Symplectic  $G$ -spaces, where Jean-Louis Koszul develops Jean-Marie Souriau's tools related to the non-equivariant case of co-adjoint action on Souriau's moment map through Souriau's Cocycle, opening the door to Lie Group Machine Learning with Souriau-Fisher metric.

This book is an introduction to modern methods of symplectic topology. It is devoted to explaining the solution of an important problem originating from classical mechanics: the 'Arnold conjecture', which asserts that the number of 1-periodic trajectories of a non-degenerate Hamiltonian system is bounded below by the dimension of the homology of the underlying manifold. The first part is a thorough introduction to Morse theory, a fundamental tool of differential topology. It defines the Morse complex and the Morse homology, and develops some of their applications. Morse homology also serves a simple model for Floer homology, which is covered in the second part. Floer homology is an infinite-dimensional analogue of Morse homology. Its involvement has been crucial in the recent achievements in symplectic geometry and in particular in the proof of the Arnold conjecture. The building blocks of Floer homology are more intricate and imply the use of more sophisticated analytical methods, all of which are explained in this second part. The three appendices present a few prerequisites in differential geometry, algebraic topology and analysis. The book

originated in a graduate course given at Strasbourg University, and contains a large range of figures and exercises. Morse Theory and Floer Homology will be particularly helpful for graduate and postgraduate students.

This text on contact topology is a comprehensive introduction to the subject, including recent striking applications in geometric and differential topology: Eliashberg's proof of Cerf's theorem via the classification of tight contact structures on the 3-sphere, and the Kronheimer-Mrowka proof of property P for knots via symplectic fillings of contact 3-manifolds. Starting with the basic differential topology of contact manifolds, all aspects of 3-dimensional contact manifolds are treated in this book. One notable feature is a detailed exposition of Eliashberg's classification of overtwisted contact structures. Later chapters also deal with higher-dimensional contact topology. Here the focus is on contact surgery, but other constructions of contact manifolds are described, such as open books or fibre connected sums. This book serves both as a self-contained introduction to the subject for advanced graduate students and as a reference for researchers.

Symplectic Geometry and Fourier Analysis

Symplectic Techniques in Physics

Riemannian Geometry of Contact and Symplectic Manifolds

Second Edition

Geometric Optics on Phase Space

This is a short tract on the essentials of differential and symplectic geometry together with a basic introduction to several applications of this rich framework: analytical mechanics, the calculus of variations, conjugate points & Morse index, and other physical topics. A central feature is the systematic utilization of Lagrangian submanifolds and their Maslov-Hörmander generating functions. Following this line of thought, first introduced by Wlodemierz Tulczyjew, geometric solutions of Hamilton-Jacobi equations, Hamiltonian vector fields and canonical transformations are described by suitable Lagrangian submanifolds belonging to distinct well-defined symplectic structures. This unified point of view has been particularly fruitful in symplectic topology, which is the modern Hamiltonian environment for the calculus of variations, yielding sharp sufficient existence conditions. This line of investigation was initiated by Claude Viterbo in 1992; here, some primary consequences of this theory are exposed in Chapter 8: aspects of Poincaré's last geometric theorem and the Arnol'd conjecture are introduced. In Chapter 7 elements of the global asymptotic treatment of the highly oscillating integrals for the Schrödinger equation are discussed: as is well known, this eventually leads to the theory of Fourier Integral Operators. This short handbook is directed toward graduate students in Mathematics and Physics and to all those who desire a quick introduction to these beautiful subjects.

This book is an introduction to semisimple Lie algebras; concise and informal, with numerous exercises and examples.

This is a book on symplectic topology, a rapidly developing field of mathematics which originated as a geometric tool for problems of classical mechanics. Since the 1980s, powerful methods such as Gromov's pseudo-holomorphic curves and Morse-Floer theory on loop spaces gave rise to the discovery of unexpected symplectic phenomena. The present book focuses on function spaces associated with a symplectic manifold. A number of recent advances show that these spaces exhibit intriguing properties and structures, giving rise to an alternative intuition and new tools in symplectic topology. The book provides an essentially self-contained introduction into these developments along with applications to symplectic topology, algebra and geometry of symplectomorphism groups, Hamiltonian dynamics and quantum mechanics. It will appeal to researchers and students from the graduate level onwards. I like the spirit of this book. It formulates concepts clearly and explains the relationship between them. The subject matter is important and interesting.

--Dusa McDuff, Barnard College, Columbia University This is a very important book, coming at the right moment. The book is a remarkable mix of introductory chapters and research topics at the very forefront of actual research. It is full of cross fertilizations of different theories, and will be useful to Ph.D. students and researchers in symplectic geometry as well as to many researchers in other fields (geometric group theory, functional analysis, mathematical quantum mechanics). It is also perfectly suited for a Ph.D.-students seminar. --Felix Schlenk, Universite de Neuchatel

The seminar Symplectic Geometry at the University of Berne in summer 1992 showed that the topic of this book is a very active field, where many different branches of mathematics come together: differential geometry, topology, partial differential equations, variational calculus, and complex analysis. As usual in such a situation, it may be tedious to collect all the necessary ingredients. The present book is intended to give the nonspecialist a solid introduction to the recent developments in symplectic and contact geometry. Chapter 1 gives a review of the symplectic group  $Sp(n, \mathbb{R})$ , symplectic manifolds, and Hamiltonian systems (last but not least to fix the notations). The Maslov index for closed curves as well as arcs in  $Sp(n, \mathbb{R})$  is discussed. This index will be used in chapters 5 and 8. Chapter 2 contains a more detailed account of symplectic manifolds starting with a proof of the Darboux theorem saying that there are no local invariants in symplectic geometry. The most important examples of symplectic manifolds will be introduced: cotangent spaces and Kahler manifolds. Finally we discuss the theory of coadjoint orbits and the Kostant-Souriau theorem, which are concerned with the question of which homogeneous spaces carry a symplectic structure.

The Restricted Three-Body Problem and Holomorphic Curves

An Introduction to Contact Topology

Holomorphic Curves in Symplectic Geometry

An Introduction to Symplectic Geometry, Hamilton Systems, and Complex Geometry

Introduction to Symplectic Topology

Book endorsed by the Sunyer Prize Committee (A. Weinstein, J. Oesterle et. al.).

Suitable for graduate students in mathematics, this monograph covers differential and symplectic geometry, homogeneous symplectic manifolds, Fourier analysis, metaplectic representation, quantization, Kirillov theory. Includes Appendix on Quantum Mechanics by Robert Hermann. 1977 edition.

A development of the basic theory and applications of mechanics with an emphasis on the role of symmetry. The book includes numerous specific applications, making it beneficial to physicists and engineers. Specific examples and applications show how the theory works, backed by up-to-date techniques, all of which make the text accessible to a wide variety of readers, especially senior undergraduates and graduates in mathematics, physics and engineering. This second edition has been rewritten and updated for clarity throughout, with a major revamping and expansion of the exercises. Internet supplements containing additional material are also available.

Symplectic geometry is very useful for clearly and concisely formulating problems in classical physics and also for

**understanding the link between classical problems and their quantum counterparts. It is thus a subject of interest to both mathematicians and physicists, though they have approached the subject from different view points. This is the first book that attempts to reconcile these approaches. The authors use the uncluttered, coordinate-free approach to symplectic geometry and classical mechanics that has been developed by mathematicians over the course of the last thirty years, but at the same time apply the apparatus to a great number of concrete problems. In the first chapter, the authors provide an elementary introduction to symplectic geometry and explain the key concepts and results in a way accessible to physicists and mathematicians. The remainder of the book is devoted to the detailed analysis and study of the ideas discussed in Chapter 1. Some of the themes emphasized in the book include the pivotal role of completely integrable systems, the importance of symmetries, analogies between classical dynamics and optics, the importance of symplectic tools in classical variational theory, symplectic features of classical field theories, and the principle of general covariance. This work can be used as a textbook for graduate courses, but the depth of coverage and the wealth of information and application means that it will be of continuing interest to, and of lasting significance for mathematicians and mathematically minded physicists.**

**Differential Geometry and Mathematical Physics**

**Elementary Symplectic Topology and Mechanics**

**First Steps in Differential Geometry**

**Introduction to Symplectic Geometry**

**Morse Theory and Floer Homology**

Over the last number of years powerful new methods in analysis and topology have led to the development of the modern global theory topology, including several striking and important results. The first edition of Introduction to Symplectic Topology was published in 1995 first comprehensive introduction to the subject and became a key text in the area. A significantly revised second edition was published in new sections and updates on the fast-developing area. This new third edition includes updates and new material to bring the book right Symplectic Geometry focuses on the processes, methodologies, and numerical approaches involved in symplectic geometry. The book first information on the symplectic and discontinuous groups, symplectic metric, and hermitian forms. Numerical calculations are presented to and transformations of these groups. The text then examines the fundamental domain of the modular group and the volume of the fund the modular group. Equations and matrices are provided to show the fundamental domain and volume of the fundamental domain of the The publication ponders on commensurable groups and unit groups of quinary quadratic forms. Numerical analyses are also offered to sh and characteristics of commensurable and unit groups. The text is a helpful reference for researchers interested in symplectic geometry The material and references in this extended second edition of "The Topology of Torus Actions on Symplectic Manifolds", published as Vo series in 1991, have been updated. Symplectic manifolds and torus actions are investigated, with numerous examples of torus actions, fo moduli spaces. Although the book is still centered on convexity results, it contains much more material, in particular lots of new example The present work grew out of a study of the Maslov class (e. g. (37]), which is a fundamental invariant in asymptotic analysis of partial equations of quantum physics. One of the many in terpretations of this class was given by F. Kamber and Ph. Tondeur (43], and it indica class is a secondary characteristic class of a complex trivial vector bundle endowed with a real reduction of its structure group. (In the Arnold about the Maslov class (2], it is also pointed out without details that the Maslov class is characteristic in the category of vector pre viously. ) Accordingly, we wanted to study the whole range of secondary characteristic classes involved in this interpretation, and w description of the results in (83]. It turned out that a complete exposition of this theory was rather lengthy, and, moreover, I felt that I would have to use a lot of scattered references in order to find the necessary information from either symplectic geometry or the theo characteristic classes. On the otherhand, both these subjects are of a much larger interest in differential geome try and topology, and in physical theories.

Symplectic Geometric Algorithms for Hamiltonian Systems

Symplectic Geometry of Affine Complex Manifolds

Lectures on Symplectic Manifolds

Function Theory on Symplectic Manifolds

Symplectic Geometry of Integrable Hamiltonian Systems

**The theory of persistence modules originated in topological data analysis and became an active area of research in algebraic topology. This book provides a concise and self-contained introduction to persistence modules and focuses on their interactions with pure mathematics, bringing the reader to the cutting edge of current research. In particular, the authors present applications of persistence to symplectic topology, including the geometry of symplectomorphism groups and embedding problems. Furthermore, they discuss topological function theory, which provides new insight into oscillation of functions. The book is accessible to readers with a basic background in algebraic and differential topology.**

The first six sections of these notes contain a description of some of the basic constructions and results on symplectic manifolds and lagrangian submanifolds. Section 7, on intersections of largrangian submanifolds, is still mostly internal to symplectic geometry, but it contains some applications to machanics and dynamical systems. Sections 8, 9, and 10 are devoted to various aspects of the quantization problem. In Section 10 there is a feedback of ideas from quantization theory into symplectic geometry itslef.

This book provides an introduction to symplectic field theory, a new and important subject which is currently being developed. The starting point of this theory are compactness results for holomorphic curves established in the last decade. The author presents a systematic introduction providing a lot of background material, much of which is scattered throughout the literature. Since the content grew out of lectures given by the author, the main aim is to provide an entry point into symplectic field theory for non-specialists and for graduate students. Extensions of certain compactness results, which are believed to be true by the specialists but have not yet been published in the literature in detail, top off the scope of this monograph.

Analysis of an old variational principal in classical mechanics has established global periodic phenomena in Hamiltonian systems. One of the links is a class of symplectic invariants, called symplectic capacities, and these invariants are the main theme of this book. Topics covered include basic symplectic geometry, symplectic capacities and rigidity, symplectic fixed point theory, and a survey on Floer homology and symplectic homology.

Introduction to Symplectic Dirac Operators

Lectures on Poisson Geometry

Symplectic Geometry and Secondary Characteristic Classes

Topological Persistence in Geometry and Analysis

Symplectic Geometry and Quantum Mechanics

**This volume is the first one that gives a systematic and self-contained introduction to the theory of symplectic Dirac operators and reflects the current state of the subject. At the same time, it is intended to establish the idea that symplectic spin geometry and symplectic Dirac operators may give valuable tools in symplectic geometry and symplectic**

topology, which have become important fields and very active areas of mathematical research.

Starting from an undergraduate level, this book systematically develops the basics of • Calculus on manifolds, vector bundles, vector fields and differential forms, • Lie groups and Lie group actions, • Linear symplectic algebra and symplectic geometry, • Hamiltonian systems, symmetries and reduction, integrable systems and Hamilton–Jacobi theory. The topics listed under the first item are relevant for virtually all areas of mathematical physics. The second and third items constitute the link between abstract calculus and the theory of Hamiltonian systems. The last item provides an introduction to various aspects of this theory, including Morse families, the Maslov class and caustics. The book guides the reader from elementary differential geometry to advanced topics in the theory of Hamiltonian systems with the aim of making current research literature accessible. The style is that of a mathematical textbook, with full proofs given in the text or as exercises. The material is illustrated by numerous detailed examples, some of which are taken up several times for demonstrating how the methods evolve and interact.

The aim of the book is to treat all three basic theories of physics, namely, classical mechanics, statistical mechanics, and quantum mechanics from the same perspective, that of symplectic geometry, thus showing the unifying power of the symplectic geometric approach. Reading this book will give the reader a deep understanding of the interrelationships between the three basic theories of physics. This book is addressed to graduate students and researchers in mathematics and physics who are interested in mathematical and theoretical physics, symplectic geometry, mechanics, and (geometric) quantization.

This second edition continues to serve as the definitive source of information about some areas of differential topology ( $J$ -holomorphic curves) and applications to quantum cohomology. The main goal of the book is to establish the fundamental theorems of the subject in full and rigorous detail. It may also serve as an introduction to current work in symplectic topology. The second edition clarifies various arguments, includes some additional results, and updates the references to recent developments.

#### Geometric Quantization

#### An Introduction to Compactness Results in Symplectic Field Theory

#### Part I. Manifolds, Lie Groups and Hamiltonian Systems

#### $J$ -holomorphic Curves and Symplectic Topology

#### The Topology of Torus Actions on Symplectic Manifolds

This book presents a survey of the geometric quantization theory of Kostant and Souriau and was first published in 1980. It has been extensively rewritten and brought up to date, with the addition of many new examples.

.  $E \subset C$ ,  $O \cong \mathbb{R}^1$ , and  $n \in \mathbb{Z}$ ,  $n \geq 2$ . Let  $\tilde{G}$  be the  $O$ -dimensional Lie  $n$  group generated by the transformation  $z \mapsto z + a$ ,  $z \in C - \{a\}$ . Then (cf.

The book serves as an introduction to holomorphic curves in symplectic manifolds, focusing on the case of four-dimensional symplectizations and symplectic cobordisms, and their applications to celestial mechanics. The authors study the restricted three-body problem using recent techniques coming from the theory of pseudo-holomorphic curves. The book starts with an introduction to relevant topics in symplectic topology and Hamiltonian dynamics before introducing some well-known systems from celestial mechanics, such as the Kepler problem and the restricted three-body problem. After an overview of different regularizations of these systems, the book continues with a discussion of periodic orbits and global surfaces of section for these and more general systems. The second half of the book is primarily dedicated to developing the theory of holomorphic curves - specifically the theory of fast finite energy planes - to elucidate the proofs of the existence results for global surfaces of section stated earlier. The book closes with a chapter summarizing the results of some numerical experiments related to finding periodic orbits and global surfaces of sections in the restricted three-body problem.

This book is also part of the Virtual Series on Symplectic Geometry

<http://www.springer.com/series/16019>

Symplectic Invariants and Hamiltonian Dynamics