Algebraic Number Theory Springer

. . . if one wants to make progress in mathematics one should study the masters not the pupils. N. H. Abel Heeke was certainly one of the masters, and in fact, the study of Heeke L series and Heeke operators has permanently embedded his name in the fabric of number theory. It is a rare occurrence when a master writes a basic book, and Heeke's Lectures on the Theory of Algebraic Numbers has become a classic. To quote another master, Andre Weil: "To improve upon Heeke, in a treatment along classical lines of the theory of algebraic numbers, would be a futile and impossible task. " We have tried to remain as close as possible to the original text in pre serving Heeke's rich, informal style of exposition. In a very few instances we have substituted modern terminology for Heeke's, e. g. , "torsion free group" for "pure group. " One problem for a student is the lack of exercises in the book. However, given the large number of texts available in algebraic number theory, this is not a serious drawback. In particular we recommend Number Fields by D. A. Marcus (Springer-Verlag) as a particularly rich source. We would like to thank James M. Vaughn Jr. and the Vaughn Foundation Fund for their encouragement and generous support of Jay R. Goldman without which this translation would never have appeared. Minneapolis George U. Brauer July 1981 Jay R.
Number Theory is more than a comprehensive treatment of the subject. It is an introduction to topics in higher level mathematics, and unique in its scope; topics from analysis, modern algebra, and discrete mathematics are all included. The book is divided into two parts. Part A covers key concepts of number theory and could serve as a first course on the subject. Part B delves into more advanced topics and an exploration of related mathematics. The prerequisites for this self-contained text are elements from linear algebra. Valuable references for the reader are collected at the end of each chapter. It is suitable as an introduction to higher level mathematics for undergraduates, or for self-study.
The problems are systematically arranged to reveal the evolution of concepts and ideas of the subject Includes various levels of problems - some are easy and straightforward, while others are more challenging All problems are elegantly solved
This undergraduate textbook provides an approachable and thorough introduction to the topic of algebraic number theory, taking the reader from unique factorisation in the integers through to the modern-day number field sieve. The first few chapters consider the importance of arithmetic in fields larger than the rational numbers. Whilst some results generalise well, the unique factorisation of the integers in these more general number fields often fail. Algebraic number theory aims to overcome this problem. Most examples are taken from quadratic fields, for which calculations are easy to perform. The middle section considers more general theory and results for number fields, and the book concludes with some topics which are more likely to be suitable for advanced students, namely, the analytic class number formula and the number field sieve. This is the first time that the number field sieve has been considered in a textbook at this level.
Early in the development of number theory, it was noticed that the ring of integers has many properties in common with the ring of polynomials over a finite field. The first part of this book illustrates this relationship by presenting analogues of various theorems. The later chapters probe the analogy between global function fields and algebraic number fields. Topics include the ABC-conjecture, Brumer-Stark conjecture, and Drinfeld modules.
A Classical Introduction to Modern Number Theory
Algebraic Theory of Numbers
Linear Algebraic Groups
Geometry, Algebra, Number Theory, and Their Information Technology Applications
TRANS19 – Transient Transcendence in Transylvania, Bra?ov, Romania, May 13–17, 2019, Revised and Extended Contributions
Volume 62 of the Encyclopedia presents the main structures and results of algebraic number theory with emphasis on algebraic number fields and class field theory. Written for the nonspecialist, the author assumes a general understanding of modern algebra and elementary number theory. Only the general properties of algebraic number fields and relate.
Itpzf}JlOV, li~oxov uoq>ZUJlCJ. 7:WV Al(JX., llpoj1. AE(Jj1. The first part of this volume is based on a course taught at Princeton University in 1961-62; at that time, an excellent set ofnotes was prepared by David Cantor, and it was originally my intention to make these notes available to the mathematical public with only quite minor changes. Then, among some old papers of mine, I accidentally came acros a long-forgotten manuscript by ChevaIley, of pre-war vintage (forgotten, that is to say, both by me and by its author) which, to my taste at least, seemed to have aged very welt It contained abrief but essentially com plete account of the main features of c1assfield theory, both local and global; and it soon became obvious that the usefulness of the intended volume would be greatly enhanced if I inc1uded such a treatment of this topic. It had to be expanded, in accordance with my own plans, but its outline could be preserved without much change. In fact, I have adhered to it rather c10sely at some critical points.
From the reviews: "...a fine book [...] When it appeared in 1949 it was a pioneer. Now there are plenty of competing accounts. But Hasse has something extra to offer.[...] Hasse proved that miracles do happen in his five beautiful papers on quadratic forms of 1923-1924. [...]It is trite but true: Every number-theorist should have this book on his or her shelf." --Irving Kaplansky in Bulletin of the American Mathematical Society, 1981
The exposition of the classical theory of algebraic numbers is clear and thorough, and there is a large number of exercises as well as worked out numerical examples. A careful study of this book will provide a solid background to the learning of more recent topics.
This is a second edition of Lang's well-known textbook. It covers all of the basic material of classical algebraic number theory, giving the student the background necessary for the study of further topics in algebraic number theory, such as cyclotomic fields, or modular forms. "Lang's books are always of great value for the graduate student and the research mathematician. This updated edition of Algebraic number theory is no exception."—-MATHEMATICAL REVIEWS
A Course in Computational Algebraic Number Theory
Classical Theory of Algebraic Numbers
Toronto, Canada, June, 2016, and Kozhikode, India, August, 2016
An Introduction via the Distribution of Primes
Quadratic Number Fields
To Number Theory Translated from the Chinese by Peter Shiu With 14 Figures Springer-Verlag Berlin Heidelberg New York 1982 HuaLooKeng Institute of Mathematics Academia Sinica Beijing The People's Republic of China PeterShlu Department of Mathematics University of Technology Loughborough Leicestershire LE 11 3 TU United Kingdom ISBN -13 : 978-3-642-68132-5 e-ISBN -13 : 978-3-642-68130-1 DOI: 10.1007/978-3-642-68130-1 Library of Congress Cataloging in Publication Data. Hua, Loo-Keng, 1910 -. Introduc tion to number theory. Translation of: Shu lun tao yin. Bibliography: p. Includes index. 1. Numbers, Theory of. I. Title. QA241.H7513.5 12'.7.82-645. ISBN-13:978-3-642-68132-5 (U.S.). AACR2 This

work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically those of translation, reprinting, reuse of illustra tions, broadcasting, reproductiOli by photocopying machine or similar means, and storage in data banks. Under {sect} 54 of the German Copyright Law where copies are made for other than private use a fee is payable to "VerwertungsgeselIschaft Wort", Munich. © Springer-Verlag Berlin Heidelberg 1982 Softcover reprint of the hardcover 1st edition 1982 Typesetting: Buchdruckerei Dipl.-Ing. Schwarz' Erben KG, Zwettl. 214113140-5432 I 0 Preface to the English Edition The reasons for writing this book have already been given in the preface to the original edition and it suffices to append a few more points

Written by an authority with great practical and teaching experience in the field, this book addresses a number of topics in computational number theory. Chapters one through five form a homogenous subject matter suitable for a six-month or year-long course in computational number theory. The subsequent chapters deal with more miscellaneous subjects.

The first edition of this book presented the theory of linear algebraic groups over an algebraically closed field. The second edition, thoroughly revised and expanded, extends the theory over arbitrary fields, which are not necessarily algebraically closed. It thus represents a higher aim. As in the first edition, the book includes a self-contained treatment of the prerequisites from algebraic geometry and commutative algebra, as well as basic results on reductive groups. As a result, the first part of the book can well serve as a text for an introductory graduate course on linear algebraic groups.

An undergraduate-level introduction to number theory, with the emphasis on fully explained proofs and examples. Exercises, together with their solutions are integrated into the text, and the first few chapters assume only basic school algebra. Elementary ideas about groups and rings are then used to study groups of units, quadratic residues and arithmetic functions with applications to enumeration and cryptography. The final part, suitable for third-year students, uses ideas from algebra, analysis, calculus and geometry to study Dirichlet series and sums of squares. In particular, the last chapter gives a concise account of Fermat's Last Theorem, from its origin in the ancient Babylonian and Greek study of Pythagorean triples to its recent proof by Andrew Wiles.

This proceedings volume gathers together original articles and survey works that originate from presentations given at the conference Transient Transcendence in Transylvania, held in Bra?ov, Romania, from May 13th to 17th, 2019. The conference gathered international experts from various fields of mathematics and computer science, with diverse interests and viewpoints on transcendence. The covered topics are related to algebraic and transcendental aspects of special functions and special numbers arising in algebra, combinatorics, geometry and number theory. Besides contributions on key topics from invited speakers, this volume also brings selected papers from attendees.

Algebraic Number Theory
A Journey Through The Realm of Numbers
Advanced Topics in Computational Number Theory
Number Fields
An Introduction to Mathematics

A description of 148 algorithms fundamental to number-theoretic computations, in particular for computations related to algebraic number theory, elliptic curves, primality testing and factoring. The first seven chapters guide readers to the heart of current research in computational algebraic number theory, including recent algorithms for computing class groups and units, as well as elliptic curve computations, while the last three chapters survey factoring and primality testing methods, including a detailed description of the number field sieve algorithm. The whole is rounded off with a description of available computer packages and some useful tables, backed by numerous exercises. Written by an authority in the field, and one with great practical and teaching experience, this is certain to become the standard and indispensable reference on the subject.

This two-volume book is a modern introduction to the theory of numbers, emphasizing its connections with other branches of mathematics. Part A is accessible to first-year undergraduates and deals with elementary number theory. Part B is more advanced and gives the reader an idea of the scope of mathematics today. The connecting theme is the theory of numbers. By exploring its many connections with other branches a broad picture is obtained. The book contains a treasury of proofs, several of which are gems seldom seen in number theory books.

A translation of Hilberts "Theorie der algebraischen Zahlkörper" best known as the "Zahlbericht", first published in 1897, in which he provides an elegantly integrated overview of the development of algebraic number theory up to the end of the nineteenth century. The Zahlbericht also provided a firm foundation for further research in the theory, and can be seen as the starting point for all twentieth century investigations into the subject, as well as reciprocity laws and class field theory. This English edition further contains an introduction by F. Lemmermeyer and N. Schappacher.

Now in its second edition, this textbook provides an introduction and overview of number theory based on the density and properties of the prime numbers. This unique approach offers both a firm background in the standard material of number theory, as well as an overview of the entire discipline. All of the essential topics are covered, such as the fundamental theorem of arithmetic, theory of congruences, quadratic reciprocity, *arithmetic functions, and the distribution of primes. New in this edition are coverage of p-adic numbers, Hensel's lemma, multiple zeta-values, and elliptic curve methods in primality testing. Key topics and features include: A solid introduction to analytic number theory, including full proofs of Dirichlet's Theorem and the Prime Number Theorem Concise treatment of algebraic number theory, including a complete presentation of primes, prime factorizations in algebraic number fields, and unique factorization of ideals Discussion of the AKS algorithm, which shows that primality testing is one of polynomial time, a topic not usually included in such texts Many interesting ancillary topics, such as primality testing and cryptography, Fermat and Mersenne numbers, and Carmichael numbers The user-friendly style, historical context, and wide range of exercises that range from simple to quite difficult (with solutions and hints provided for select exercises) make Number Theory: An Introduction via the Density of Primes ideal for both self-study and classroom use. Intended for upper level undergraduates and beginning graduates, the only prerequisites are a basic knowledge of calculus, multivariable calculus, and some linear algebra. All necessary concepts from abstract algebra and complex analysis are introduced where needed.*

This book is a revised and greatly expanded version of our book Elements of Number Theory published in 1972. As with the first book the primary audience we envisage consists of upper level undergraduate mathematics majors and graduate students. We have assumed some familiarity with the material in a standard undergraduate course in abstract algebra. A large portion of Chapters 1-11 can be read even without such background with the aid of a small amount of supplementary reading. The later chapters assume some knowledge of Galois theory, and in Chapters 16 and 18 an acquaintance with the theory of complex variables is necessary. Number theory is an ancient subject and its content is vast. Any intro ductory book must, of necessity, make a very limited selection from the fascinat ing array of possible topics. Our focus is on topics which point in the direction of algebraic number theory and arithmetic algebraic geometry. By a careful selection of subject matter we have found it possible to exposit some rather advanced material without requiring very much in the way oftechnical background. Most of this material is classical in the sense that is was dis covered during the nineteenth century and earlier, but it is also modern because it is intimately related to important research going on at the present time.

From Hilbert to Tate

Rational Number Theory in the 20th Century

A Course in Algebraic Number Theory

The Theory of Classical Valuations

Transcendence in Algebra, Combinatorics, Geometry and Number Theory

This book is an exposition of the main ideas of algebraic number theory. It is written for the non-expert. Therefore, beyond some algebra, there are almost no prerequisites.

This introduction to algebraic number theory discusses the classical concepts from the viewpoint of Arakelov theory. The treatment of class theory is particularly rich in illustrating complements, offering hints for further study, and providing concrete examples. It is the most up-to-date, systematic, and theoretically comprehensive textbook on algebraic number field theory available.

By focusing on quadratic numbers, this advanced undergraduate or master's level textbook on algebraic number theory is accessible even to students who have yet to learn Galois theory. The techniques of elementary arithmetic, ring theory and linear algebra are shown working together to prove important theorems, such as the unique factorization of ideals and the finiteness of the ideal class group. The book concludes with two topics particular to quadratic fields: continued fractions and quadratic forms. The treatment of quadratic forms is somewhat more advanced than usual, with an emphasis on their connection with ideal classes and a discussion of Bhargava cubes. The numerous exercises in the text offer the reader hands-on computational experience with elements and ideals in quadratic number fields. The reader is also asked to fill in the details of proofs and develop extra topics, like the theory of orders. Prerequisites include elementary number theory and a basic familiarity with ring theory.

The title of this book may be read in two ways. One is 'algebraic number-theory', that is, the theory of numbers viewed algebraically; the other, 'algebraic-number theory', the study of algebraic numbers. Both readings are compatible with our aims, and both are perhaps misleading. Misleading, because a proper coverage of either topic would require more space than is available, and demand more of the reader than we wish to; compatible, because our aim is to illustrate how some of the basic notions of the theory of algebraic numbers may be applied to problems in number theory. Algebra is an easy subject to compartmentalize, with topics such as 'groups', 'rings' or 'modules' being taught in comparative isolation. Many students view it this way. While it would be easy to exaggerate this tendency, it is not an especially desirable one. The leading mathematicians of the nineteenth and early twentieth centuries developed and used most of the basic results and techniques of linear algebra for perhaps a hundred years, without ever defining an abstract vector space: nor is there anything to suggest that they suf fered thereby. This historical fact may indicate

that abstrac tion is not always as necessary as one commonly imagines; on the other hand the axiomatization of mathematics has led to enormous organizational and conceptual gains.
This volume contains proceedings of two conferences held in Toronto (Canada) and Kozhikode (India) in 2016 in honor of the 60th birthday of Professor Kumar Murty. The meetings were focused on several aspects of number theory: The theory of automorphic forms and their associated L-functions Arithmetic geometry, with special emphasis on algebraic cycles, Shimura varieties, and explicit methods in the theory of abelian varieties The emerging applications of number theory in information technology Kumar Murty has been a substantial influence in these topics, and the two conferences were aimed at honoring his many contributions to number theory, arithmetic geometry, and information technology.
An Introduction via the Density of Primes
The Theory of Algebraic Number Fields
Fermat's Last Theorem

Lectures on the Theory of Algebraic Numbers

Algebraic number theory introduces students not only to new algebraic notions but also to related concepts: groups, rings, fields, ideals, quotient rings and quotient fields, homomorphisms and isomorphisms, modules, and vector spaces. Author Pierre Samuel notes that students benefit from their studies of algebraic number theory by encountering many concepts fundamental to other branches of mathematics — algebraic geometry, in particular. This book assumes a knowledge of basic algebra but supplements its teachings with brief, clear explanations of integrality, algebraic extensions of fields, Galois theory, Noetherian rings and modules, and rings of fractions. It covers the basics, starting with the divisibility theory in principal ideal domains and ending with the unit theorem, finiteness of the class number, and the more elementary theorems of Hilbert ramification theory. Numerous examples, applications, and exercises appear throughout the text.
Computational algebraic number theory has been attracting broad interest in the last few years due to its potential applications in coding theory and cryptography. For this reason, the Deutsche Mathematiker-Vereinigung initiated an introductory graduate seminar on this topic in Dusseldorf. The lectures given there by the author served as the basis for this book which allows fast access to the state of the art in this area. Special emphasis has been placed on practical algorithms - all developed in the last five years - for the computation of integral bases, the unit group and the class group of arbitrary algebraic number fields. The workshops organized by the Gesselschaft fur mathematische Forschung in cooperation with the Deutsche Mathematiker-Vereinigung (German Mathematics Society) are intended to help, in particular, students and younger mathematicians, to obtain an introduction to fields of current research. Through the means of these well-organized seminars, scientists from other fields can also be introduced to new mathematical ideas. The publication of these workshops in the series DMV SEMINAR will make the material available to an even larger audience.
This introduction to algebraic number theory via the famous problem of "Fermats Last Theorem" follows its historical development, beginning with the work of Fermat and ending with Kummers theory of "ideal" factorization. The more elementary topics, such as Eulers proof of the impossibilty of x+y=z, are treated in an uncomplicated way, and new concepts and techniques are introduced only after having been motivated by specific problems. The book also covers in detail the application of Kummers theory to quadratic integers and relates this to Gauss'theory of binary quadratic forms, an interesting and important connection that is not explored in any other book.
This undergraduate textbook provides an elegant introduction to the arithmetic of quadratic number fields, including many topics not usually covered in books at this level. Quadratic fields offer an introduction to algebraic number theory and some of its central objects: rings of integers, the unit group, ideals and the ideal class group. This textbook provides solid grounding for further study by placing the subject within the greater context of modern algebraic number theory. Going beyond what is usually covered at this level, the book introduces the notion of modularity in the context of quadratic reciprocity, explores the close links between number theory and geometry via Pell conics, and presents applications to Diophantine equations such as the Fermat and Catalan equations as well as elliptic curves. Throughout, the book contains extensive historical comments, numerous exercises (with solutions), and pointers to further study. Assuming a moderate background in elementary number theory and abstract algebra, Quadratic Number Fields offers an engaging first course in algebraic number theory, suitable for upper undergraduate students.
This book takes the reader on a journey from familiar high school mathematics to undergraduate algebra and number theory. The journey starts with the basic idea that new number systems arise from solving different equations, leading to (abstract) algebra. Along this journey, the reader will be exposed to important ideas of mathematics, and will learn a little about how mathematics is really done. Starting at an elementary level, the book gradually eases the reader into the complexities of higher mathematics; in particular, the formal structure of mathematical writing (definitions, theorems and proofs) is introduced in simple terms. The book covers a range of topics, from the very foundations (numbers, set theory) to basic abstract algebra (groups, rings, fields), driven throughout by the need to understand concrete equations and problems, such as determining which numbers are sums of squares. Some topics usually reserved for a more advanced audience, such as Eisenstein integers or quadratic reciprocity, are lucidly presented in an accessible way. The book also introduces the reader to open source software for computations, to enhance understanding of the material and nurture basic programming skills. For the more adventurous, a number of Outlooks included in the text offer a glimpse of possible mathematical excursions. This book supports readers in transition from high school to university mathematics, and will also benefit university students keen to explore the beginnings of algebraic number theory. It can be read either on its own or as a supporting text for first courses in

algebra or number theory, and can also be used for a topics course on Diophantine equations.
From PNT to FLT
Problems in Algebraic Number Theory
An Introduction to Mathematics: Part B
Elements of Number Theory
From Quadratic Equations to Quadratic Reciprocity

Valuation theory is used constantly in algebraic number theory and field theory, and is currently gaining considerable research interest. Ribenboim fills a unique niche in the literature as he presents one of the first introductions to classical valuation theory in this up-to-date rendering of the authors long-standing experience with the applications of the theory. The presentation is fully up-to-date and will serve as a valuable resource for students and mathematicians.

Algebraic Number TheorySpringer

This book deals with several aspects of what is now called "explicit number theory." The central theme is the solution of Diophantine equations, i.e., equations or systems of polynomial equations which must be solved in integers, rational numbers or more generally in algebraic numbers. This theme, in particular, is the central motivation for the modern theory of arithmetic algebraic geometry. In this text, this is considered through three of its most basic aspects. The local aspect, global aspect, and the third aspect is the theory of zeta and L-functions. This last aspect can be considered as a unifying theme for the whole subject.

This book provides an introduction and overview of number theory based on the distribution and properties of primes. This unique approach provides both a firm background in the standard material as well as an overview of the whole discipline. All the essential topics are covered: fundamental theorem of arithmetic, theory of congruences, quadratic reciprocity, arithmetic functions, and the distribution of primes. Analytic number theory and algebraic number theory both receive a solid introductory treatment. The book's user-friendly style, historical context, and wide range of exercises make it ideal for self study and classroom use.

This book is divided into two parts. The first part is preliminary and consists of algebraic number theory and the theory of semisimple algebras. There are two principal topics: classification of quadratic forms and quadratic Diophantine equations. The second topic is a new framework which contains the investigation of Gauss on the sums of three squares as a special case. To make the book concise, the author proves some basic theorems in number theory only in some special cases. However, the book is self-contained when the base field is the rational number field, and the main theorems are stated with an arbitrary number field as the base field. So the reader familiar with class field theory will be able to learn the arithmetic theory of quadratic forms with no further references.

A Genetic Introduction to Algebraic Number Theory
The Story of Algebraic Numbers in the First Half of the 20th Century
Number Theory in Function Fields
Elementary Number Theory
Volume II: Analytic and Modern Tools

Solutions of equations in integers is the central problem of number theory and is the focus of this book. The amount of material is suitable for a one-semester course. The author has tried to avoid the ad hoc proofs in favor of unifying ideas that work in many situations. There are exercises at the end of almost every section, so that each new idea or proof receives immediate reinforcement.

The last one hundred years have seen many important achievements in the classical part of number theory. After the proof of the Prime Number Theorem in 1896, a quick development of analytical tools led to the invention of various new methods, like Brun's sieve method and the circle method of Hardy, Littlewood and Ramanujan; developments in topics such as prime and additive number theory, and the solution of Fermat's problem. Rational Number Theory in the 20th Century: From PNT to FLT offers a short survey of 20th century developments in classical number theory, documenting between the proof of the Prime Number Theorem and the proof of Fermat's Last Theorem. The focus lays upon the part of number theory that deals with properties of integers and rational numbers. Chapters are divided into five time periods, which are then further divided into subject areas. With the introduction of each new topic, developments are followed through to the present day. This book will appeal to graduate researchers and student in number theory, however the presentation of main results without technicalities will make this accessible to anyone with an interest in the area.

This text for a graduate-level course covers the general theory of factorization of ideals in Dedekind domains as well as the number field case. It illustrates the use of Kummer's theorem, proofs of the Dirichlet unit theorem, and Minkowski bounds on element and ideal norms. 2003 edition.

Requiring no more than a basic knowledge of abstract algebra, this text presents the mathematics of number fields in a straightforward, pedestrian manner. It therefore avoids local methods and presents proofs in a way that highlights the important parts of the arguments. Readers are assumed to be able to fill in the details, which in many places are left as exercises.

From the reviews: "... The author succeeded in an excellent way to describe the various points of view under which Class Field Theory can be seen. ... In any case the author succeeded to write a very readable book on these difficult themes." Monatshefte fuer Mathematik, 1994 "... Number theory is not easy and quite technical at several places, as the author is able to show in his technically good exposition. The amount of difficult material well exposed gives a survey of quite a lot of good solid classical number theory... Conclusion: for people not already familiar with this field this book is not so easy to read, but for the specialist in number theory this is a useful description of (classical) algebraic number theory." Medelingen van het wiskundig genootschap, 1995

Number Theory II
Arithmetic of Quadratic Forms
Basic Number Theory.
Algebraic Theory of Quadratic Numbers
Number Theory

The book is aimed at people working in number theory or at least interested in this part of mathematics. It presents the development of the theory of algebraic numbers up to the year 1950 and contains a rather complete bibliography of that period. The reader will get information about results obtained before 1950. It is hoped that this may be helpful in preventing rediscoveries of old results, and might also inspire the reader to look at the work done earlier, which may hide some ideas which could be applied in contemporary research.

Introduction to Number Theory
Computational Algebraic Number Theory