Algebraic Geometry I Algebraic Curves Algebraic Manifolds And Schemes

This book provides an accessible and self-contained introduction to the theory of algebraic curves over a finite field, a subject that has been of fundamental importance to mathematics for many years and that has essential applications in areas such as finite geometry, number theory, error-correcting codes, and cryptology. Unlike other books, this one emphasizes the algebraic g function field approach to algebraic curves. The authors begin by developing the general theory of curves over any field, highlighting peculiarities occurring for positive characteristic and requiring of the reader only basic knowledge of algebra and geometry. The special properties that a curve over a finite field can have are then discussed. The geometrical theory of linear series i for the number of rational points on a curve, following the theory of Stöhr and Voloch. The approach of Hasse and Weil via zeta functions is explained, and then attention turns to more advanced results: a state-of-the-art introduction to maximal curves over finite fields is provided; a comprehensive account is given of the automorphism group of a curve; and some applications to geometry are described. The book includes many examples and exercises. It is an indispensable resource for researchers and the ideal textbook for graduate students.

This is an excellent introduction to algebraic geometry, which assumes only standard undergraduate mathematical topics: complex analysis, rings and fields, and topology. Reading this book will help establish the geometric intuition that lies behind the more advanced ideas and techniques used in the study of higher-dimensional varieties.

Conics and Cubics offers an accessible and well illustrated introduction to algebraic curves. By classifying irreducible cubics over the real numbers and proving that their points form Abelian groups, the book gives readers easy access to the study of elliptic curves. It includes a simple proof of Bezout's Theorem on the number of intersections of two curves. The subject area is de concrete and accessible examples. The book is a text for a one-semester course.

This book is about modern algebraic geometry. The title A Royal Road to Algebraic Geometry is inspired by the famous anecdote about the king asking Euclid if there really existed no simpler way for learning geometry, than to read all of his work Elements. Euclid is said to have answered: "There is no royal road to geometry!" The book starts by explaining this enigmatic answer being to argue that indeed, in some sense there is a royal road to algebraic geometry. From a point of departure in algebraic curves, the exposition moves on to the present shape of the field, culminating with Alexander Grothendieck's theory of schemes. Contemporary homological tools are explained. The reader will follow a directed path leading up to the main elements of mod When the road is completed, the reader is empowered to start navigating in this immense field, and to open up the door to a wonderful field of research. The greatest scientific experience of a lifetime!

Algebraic Geometry and Arithmetic Curves

Algebraic Geometry

Algebraic Curves and One-Dimensional Fields

Geometry of Algebraic Curves

Author S.A. Stepanov thoroughly investigates the current state of the theory of Diophantine equations and its related methods. Discussions focus on arithmetic, algebraic-geometric, and logical aspects of the problem. Designed for students as well as researchers, the book includes over 250 excercises accompanied by hints, instructions, and references. Written in a clear manner, this text does not require readers to have special knowledge of modern methods of algebraic geometry.

Vertex algebras are algebraic objects that encapsulate the concept of operator product expansion from two-dimensional conformal field theory. Vertex algebras are fast becoming ubiquitous in many areas of modern mathematics, with applications to representation theory, algebraic geometry, the theory of finite groups, modular functions, topology, integrable systems, and combinatorics. This book is an introduction to the theory of vertex algebras with a particular emphasis on the relationship with the geometry of algebraic curves. The notion of a vertex algebra is introduced in a coordinate-independent way, so that vertex operators become well defined on arbitrary smooth algebraic curves, possibly equipped with additional data, such as a vector bundle. Vertex algebras then appear as the algebraic objects encoding the geometric structure of various moduli spaces associated with algebraic curves. Therefore they may be used to give a geometric interpretation of various questions of representation theory. The book contains many original results, introduces important new concepts, and brings new insights into the theory of vertex algebras. The authors have made a great effort to make the book self-contained and accessible to readers of all backgrounds. Reviewers of the first edition anticipated that it would have a long-lasting influence on this exciting field of mathematics and would be very useful for graduate students and researchers interested in the subject. This second edition, substantially improved and expanded, includes several new topics, in particular an introduction to the Beilinson-Drinfeld theory of factorization algebras and the geometric Langlands correspondence.

An introduction to abstract algebraic geometry, with the only prerequisites being results from commutative algebra, which are stated as needed, and some elementary topology. More than 400 exercises distributed throughout the book offer specific examples as well as more specialised topics not treated in the main text, while three appendices present brief accounts of some areas of current research. This book can thus be used as textbook for an introductory course in algebraic geometry following a basic graduate course in algebra. Robin Hartshorne studied algebraic geometry with Oscar Zariski and David Mumford at Harvard, and with J.-P. Serre and A. Grothendieck in Paris. He is the author of "Residues and Duality", "Foundations of Projective Geometry", "Ample Subvarieties of Algebraic Varieties", and numerous research titles.

Algebraic curves have many special properties that make their study particularly rewarding. As a result, curves provide a natural introduction to algebraic geometry. In this book, the authors also bring out aspects of curves that are unique to them and emphasize connections with algebra. This text covers the essential topics in the geometry of algebraic curves, such as line bundles and vector bundles, the Riemann-Roch Theorem, divisors, coherent sheaves, and zeroth and firstcohomology groups. The authors make a point of using concrete examples and explicit methods to ensure that the style is clear and understandable. Several chapters develop the connections between the geometry of algebraic curves and the algebra of one-dimensional fields. This is an interesting topic that israrely found in introductory texts on algebraic geometry. This book makes an excellent text for a first course for graduate students.

A Royal Road to Algebraic Geometry

Algebraic Geometry I

Arithmetic of Algebraic Curves

With an Appendix by Oleg Viro

The book presents the central facts of the local, projective and intrinsic theories of complex algebraic plane curves, with complete proofs and starting from low-level prerequisites. It includes Puiseux series, branches, intersection multiplicity, Bézout theorem, rational functions, Riemann-Roch theorem and rational maps. It is aimed at graduate and advanced undergraduate students, and also at anyone interested in algebraic curves or in an introduction to algebraic geometry via curves.

In recent years there has been enormous activity in the theory of algebraic curves. Many long-standing problems have been solved using the general techniques developed in algebraic geometry during the 1950's and 1960's. Additionally, unexpected and deep connections between algebraic curves and differential equations have been uncovered, and these in turn shed light on other classical problems in curve theory. It seems fair to say that the theory of algebraic curves looks completely different now from how it appeared 15 years ago; in particular, our current state of knowledge repre sents a significant advance beyond the legacy left by the classical geometers such as Noether, Castelnuovo, Enriques, and Severi. These books give a presentation of one of the central areas of this recent activity; namely, the study of linear series on both a fixed curve (Volume I) and on a variable curve (Volume II). Our goal is to give a comprehensive and self-contained account of the extrinsic geometry of algebraic curves, which in our opinion constitutes the main geometric core of the recent advances in curve theory. Along the way we shall, of course, discuss appli cations of the theory of linear series to a number of classical topics (e.g., the geometry of the Riemann theta divisor) as well as to some of the current research (e.g., the Kodaira dimension of the moduli space of curves).

The second volume of the Geometry of Algebraic Curves is devoted to the foundations of the theory of moduli of algebraic curves. Its authors are research mathematicians who have actively participated in the development of the Geometry of Algebraic Curves. The subject is an extremely fertile and active one, both within the mathematical community and at the interface with the theoretical physics community. The approach is unique in its blending of algebro-geometric, complex analytic and topological/combinatorial methods. It treats important topics such as Teichmüller theory, the cellular decomposition of moduli and its consequences and the Witten conjecture. The careful and comprehensive presentation of the material is of value to students who wish to learn the subject and to experts as a reference source. The first volume appeared 1985 as vol. 267 of the same series.

An accessible introduction to plane algebraic curves that also serves as a natural entry point to algebraic geometry.

A Computer Algebra Approach

A Guide to Plane Algebraic Curves

Towards Moduli Spaces

This book provides a self-contained introduction to the theory of error-correcting codes and related topics in number theory, Algebraic Geometry and the theory of Sphere Packings. The material is presented in an easily understandable form. This book is devoted to geometric Goppa codes; the recently discovered areas which combines Coding Theory, Algebraic Geometry, Number Theory, and Theory of Sphere Packings. It has an interdisciplinary nature and demonstrates the close interconnection of Coding Theory with various classical areas of mathematics. There are four main themes in the book. The first is a brief exposition of the basic concepts and facts of error-correcting code theory. The second is a complete presentation of the theory of algebraic curves; especially the curves defined over finite fields. The third is a detailed description of the theory of elliptic and modular codes, and their reductions modulo a prime number. The fourth is a construction of geometric Gappa codes producing rather long linear codes with very good parameters coming from algebraic curves, and with a lot of rational points. The aim of the book is to present these themes in a simple, easily understandable manner, and explain their close interconnection. At the same time the book introduces the reader to topics which are at the forefront of current research.

This book offers a comprehensive treatment of geometric analysis on symmetric spaces, with applications to representation theory. The author's thorough, accurate approach brings the reader up to date on current research in analysis on symmetric spaces and the analytic approach to the representation theory of semisimple Lie groups. The author is a 1988 Steele Prize recipient for his earlier books in this area.

This book offers a concise yet thorough introduction to the notion of moduli spaces of complex algebraic curves. Over the last few decades, this notion has become central not only in algebraic geometry, but in mathematical physics, including string theory, as well. The book begins by studying individual smooth algebraic curves, including the most beautiful ones, before addressing families of curves. Studying families of algebraic curves often proves to be more efficient than studying individual curves: these families and their total spaces can still be smooth, even if there are singular curves among their members. A major discovery of the 20th century, attributed to P. Deligne and D. Mumford, was that curves with only mild singularities form smooth compact moduli spaces. An unexpected byproduct of this discovery was the realization that the analysis of more complex curve singularities is not a necessary step in understanding the geometry of the moduli spaces. The book does not use the sophisticated machinery of modern algebraic geometry, and most classical objects related to curves – such as Jacobian, space of holomorphic differentials, the Riemann-Roch theorem, and Weierstrass points – are treated at a basic level that does not require a profound command of algebraic geometry, but which is sufficient for extending them to vector bundles and other geometric objects associated to moduli spaces. Nevertheless, it offers clear information on the construction of the moduli spaces, and provides readers with tools for practical operations with this notion. Based on several lecture courses given by the authors at the Independent University of Moscow and Higher School of Economics, the book also includes a wealth of problems, making it suitable not only for individual research, but also as a textbook for undergraduate and graduate coursework

A thorough introduction to the theory of algebraic plane curves and their relations to various fields of geometry and analysis. Almost entirely confined to the properties of the general curve, and chiefly employs algebraic procedure. Geometric methods are much employed, however, especially those involving the projective geometry of hyperspace. 1931 edition. 17 illustrations.

Algebraic Functions and Projective Curves

Plane Algebraic Curves

A Concrete Introduction to Algebraic Curves

Rational Algebraic Curves

This book gives an introduction to algebraic functions and projective curves. It covers a wide range of material by dispensing with the machinery of algebraic geometry and proceeding directly via valuation theory to the main results on function fields. It also develops the theory of singular curves by studying maps to projective space, including topics such as Weierstrass points in characteristic p, and the Gorenstein relations for singularities of plane curves.

This book contains several fundamental ideas that are revived time after time in different guises, providing a better understanding of algebraic geometric phenomena. It shows how the field is enriched with loans from analysis and topology and from commutative algebra and homological algebra.

This development of the theory of complex algebraic curves was one of the peaks of nineteenth century mathematics. They have many fascinating properties and arise in various areas of mathematics, from number theory to theoretical physics, and are the subject of much research. By using only the basic techniques acquired in most undergraduate courses in mathematics, Dr. Kirwan introduces the theory, observes the algebraic and topological properties of complex algebraic curves, and shows how they are related to complex analysis.

This introduction to algebraic geometry allows readers to grasp the fundamentals of the subject with only linear algebra and calculus as prerequisites. After a brief history of the subject, the book introduces projective spaces and projective varieties, and explains plane curves and resolution of their singularities. The volume further develops the geometry of algebraic curves and treats congruence zeta functions of algebraic curves over a finite field. It concludes with a complex analytical discussion of algebraic curves. The author emphasizes computation of concrete examples rather than proofs, and these examples are discussed from various viewpoints. This approach allows readers to develop a deeper understanding of the theorems.

Vertex Algebras and Algebraic Curves: Second Edition

Algebraic Curves and Riemann Surfaces

Introduction to Algebraic Curves

Introduction to Plane Algebraic Curves

The book was easy to understand, with many examples. The exercises were well chosen, and served to give further examples and developments of the theory. --William Goldman, University of Maryland In this book, Miranda takes the approach that algebraic curves are best encountered for the first time over the complex numbers, where the reader's classical intuition about surfaces, integration, and other concepts can be brought into play. Therefore, many examples of algebraic curves are presented in the first chapters. In this way, the book begins as a primer on Riemann surfaces, with complex charts and meromorphic functions taking center stage. But the main examples come from projective curves, and slowly but surely the text moves toward the algebraic category. Proofs of the Riemann-Roch and Serre Duality Theorems are presented in an algebraic manner, via an adaptation of the adelic proof, expressed completely in terms of solving a Mittag-Leffler problem. Sheaves and cohomology are introduced as a unifying device in the latter chapters, so that their utility and naturalness are immediately obvious. Requiring a background of one semester of complex variable theory and a year of abstract algebra, this is an excellent graduate textbook for a second-semester course in complex variables or a year-long course in algebraic geometry.

This is a graduate-level text on algebraic geometry that provides a quick and fully self-contained development of the fundamentals, including all commutative algebra which is used. A taste of the deeper theory is given: some topics, such as local algebra and ramification theory, are treated in depth. The book culminates with a selection of topics from the theory of algebraic curves, including the Riemann-Roch theorem, elliptic curves, the zeta function of a curve over a finite field, and the Riemann hypothesis for elliptic curves.

This book is a very readable introduction to algebraic geometry and will be immensely useful to mathematicians working in algebraic geometry and complex analysis and especially to graduate students in these fields.

"... To sum up, this book helps to learn algebraic geometry in a short time, its concrete style is enjoyable for students and reveals the beauty of mathematics." --Acta Scientiarum Mathematicarum

Algebraic Curves, Algebraic Manifolds and Schemes

A First Course

Algebraic Curves and One-dimensional Fields

Singular Algebraic Curves

Singular algebraic curves have been in the focus of study in algebraic geometry from the very beginning, and till now remain a subject of an active research related to many modern developments in algebraic geometry, symplectic geometry, and tropical geometry. The monograph suggests a unified approach to the geometry of singular algebraic curves on algebraic surfaces and their families, which applies to arbitrary singularities, allows o questions concerning the geometry of equisingular families of curves, and, finally, leads to results which can be viewed as the best possible in a reasonable sense. Various methods of the cohomology vanishing theory as well as the patchworking construction with its modifications will be of a special interest for experts in algebraic geometry and singularity theory. The introductory chapters on zero-dimensional schemes and global deformati serve as a material for special courses and seminars for graduate and post-graduate students.Geometry in general plays a leading role in modern mathematics, and algebraic geometry is the most advanced area of research in geometry. In turn, algebraic curves for more than one century have been the central subject of algebraic geometry both in fundamental theoretic questions and in applications to other fields of mathematics and mathe Particularly, the local and global study of singular algebraic curves involves a variety of methods and deep ideas from geometry, analysis, algebra, combinatorics and suggests a number of hard classical and newly appeared problems which inspire further development in this research area.

"This book succeeds brilliantly by concentrating on a number of core topics...and by treating them in a hugely rich and varied way. The author ensures that the reader will learn a large amount of classical material and perhaps more importantly, will also learn that there is no one approach to the subject. The essence lies in the range and interplay of possible approaches. The author is to be congratulated on a work of deep and enthusiastic s --MATHEMATICAL REVIEWS

This book is a general introduction to the theory of schemes, followed by applications to arithmetic surfaces and to the theory of reduction of algebraic curves. The first part introduces basic objects such as schemes, morphisms, base change, local properties (normality, regularity, Zariski's Main Theorem). This is followed by the more global aspect: coherent sheaves and a finiteness theorem for their cohomology groups. Then follows a chap differentials, dualizing sheaves, and Grothendieck's duality theory. The first part ends with the theorem of Riemann-Roch and its application to the study of smooth projective curves over a field. Singular curves are treated through a detailed study of the Picard group. The second part starts with blowing-ups and desingularisation (embedded or not) of fibered surfaces over a Dedekind ring that leads on to intersection theory on arithmetic surf criterion is proved and also the existence of the minimal regular model. This leads to the study of reduction of algebraic curves. The case of elliptic curves is studied in detail. The book concludes with the fundmental theorem of stable reduction of Deligne-Mumford. The book is essentially self-contained, including the necessary material on commutative algebra. The prerequisites are therefore few, and the book should suit a graduate stude examples and nearly 600 exercises.

The aim of these notes is to develop the theory of algebraic curves from the viewpoint of modern algebraic geometry, but without excessive prerequisites. We have assumed that the reader is familiar with some basic properties of rings, ideals and polynomials, such as is often covered in a one-semester course in modern algebra; additional commutative algebra is developed in later sections.

Codes on Algebraic Curves

Algebraic Geometry for Scientists and Engineers

History Algebraic Geometry

Algebraic Curves over a Finite Field

This book, based on lectures presented in courses on algebraic geometry taught by the author at Purdue University, is intended for engineers and scientists (especially computer scientists), as well as graduate students and advanced undergraduates in mathematics. In addition to providing a concrete or algorithmic approach to algebraic geometry, the author also attempts to motivate and explain its link to more modern algebraic geometry based on abstract algebra.The book covers various topics in the theory of algebraic curves and surfaces, such as rational and polynomial parametrization, functions and differentials on a curve, branches and valuations, and resolution of singularities. The emphasis is on presenting heuristic ideas and suggestive arguments rather than formal proofs. Readers will gain new insight into the subject of algebraic geometry in a way that should increase appreciation of modern treatments of the subject, as well as enhance its utility in applications in science and industry.

This volume contains a collection of papers on algebraic curves and their applications. While algebraic curves traditionally have provided a path toward modern algebraic geometry, they also provide many applications in number theory, computer security and cryptography, coding theory, differential equations, and more. Papers cover topics such as the rational torsion points of elliptic curves, arithmetic statistics in the moduli space of curves, combinatorial descriptions of semistable hyperelliptic curves over local fields, heights on weighted projective spaces, automorphism groups of curves, hyperelliptic curves, dessins d'enfants, applications to Painlevé equations, descent on real algebraic varieties, quadratic residue codes based on hyperelliptic curves, and Abelian varieties and cryptography. This book will be a valuable resource for people interested in algebraic curves and their connections to other branches of mathematics.

Algebraic Geometry IAlgebraic Curves, Algebraic Manifolds and SchemesSpringer Science & Business Media

This book differs from a number of recent books on this subject in that it combines analytic and geometric methods at the outset, so that the reader can grasp the basic results of the subject. Although such modern techniques of sheaf theory, cohomology, and commutative algebra are not covered here, the book provides a solid foundation to proceed to more advanced texts in general algebraic geometry, complex manifolds, and Riemann surfaces, as well as algebraic curves. Containing numerous exercises this book would make an excellent introductory text.

Volume II with a contribution by Joseph Daniel Harris

Algebraic Curves

Conics and Cubics

Volume I

The central problem considered in this introduction for graduate students is the determination of rational parametrizability of an algebraic curve and, in the positive case, the computation of a good rational parametrization. This amounts to determining the genus of a curve: its complete singularity structure, computing regular points of the curve in small coordinate fields, and constructing linear systems of curves with prescribed intersection multiplicities. The book discusses various optimality criteria for rational parametrizations of algebraic curves.

* Employs proven conception of teaching topics in commutative algebra through a focus on their applications to algebraic geometry, a significant departure from other works on plane algebraic curves in which the topological-analytic aspects are stressed *Requires only a basic knowledge of algebra, with all necessary algebraic facts collected into several appendices * Studies algebraic curves over an algebraically closed field K and those of prime characteristic, which can be applied to coding theory and cryptography * Covers filtered algebras, the associated graded rings and Rees rings to deduce basic facts about intersection theory of plane curves, applications of which are standard tools of computer algebra * Examples, exercises, figures and suggestions for further study round out this fairly self-contained textbook

This text covers the essential topics in the geometry of algebraic curves, such as line and vector bundles, the Riemann-Roch Theorem, divisors, coherent sheaves, and zeroth and first cohomology groups. It demonstrates how curves can act as a natural introduction to algebraic geometry.

Algebraic Curves, the Brill and Noether Way

Algebraic Curves : Algebraic Manifolds and Schemes

Algebraic Curves and Their Applications

Complex Algebraic Curves